Chapter 11
Counting Methods and Probability Theory

Check Points 11.1

1. Multiply the number of choices for each of the two courses of the meal:
   Appetizers: Main Courses:
   $10 \times 15 = 150$

2. Multiply the number of choices for each of the two courses:
   Psychology: Social Science:
   $10 \times 4 = 40$

3. Multiply the number of choices for each of the three decisions:
   Size: Crust: Topping:
   $2 \times 3 \times 5 = 30$

4. Multiply the number of choices for each of the five options:
   Color: A/C: Electric/Gas: Onboard Computer: Global Positioning System:
   $10 \times 2 \times 2 \times 2 \times 2 = 160$

5. Multiply the number of choices for each of the six questions:
   Question #1: Question #2: Question #3: Question #4: Question #5: Question #6:
   $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6 = 729$

6. Multiply the number of choices for each of the five digits:
   Digit 1: Digit 2: Digit 3: Digit 4: Digit 5:
   $9 \times 10 \times 10 \times 10 \times 10 = 90,000$

Concept and Vocabulary Check 11.1

1. $M \cdot N$

2. multiplying; Fundamental Counting

3. false

4. true

Exercise Set 11.1

1. $8 \cdot 10 = 80$

2. $9 \cdot 3 = 27$

3. $3 \cdot 4 = 12$

4. $5 \cdot 6 = 30$
5. \(3 \cdot 2 = 6\)

6. \(26 \cdot 9 = 234\)

7. Multiply the number of choices for each of the three decisions:
   \[
   \begin{array}{ccc}
   \text{Drink:} & \times & \text{Size:} & \times & \text{Flavor:} \\
   2 & & 4 & & 5 \\
   \end{array}
   = 40
   
8. Multiply the number of choices for each of the three decisions:
   \[
   \begin{array}{ccc}
   \text{Size:} & \times & \text{Crust:} & \times & \text{Topping:} \\
   3 & & 4 & & 6 \\
   \end{array}
   = 72
   
9. Multiply the number of choices for each of the four menu categories:
   \[
   \begin{array}{cccc}
   \text{Main Course:} & \times & \text{Vegetables:} & \times & \text{Beverages:} & \times & \text{Desserts:} \\
   4 & & 3 & & 4 & & 3 \\
   \end{array}
   = 144
   
   This includes, for example, an order of ham and peas with tea and cake.
   This also includes an order of beef and peas with milk and pie.

10. Multiply the number of choices for each of the four apartment options:
    \[
    \begin{array}{cccc}
    \text{Option A:} & \times & \text{Option B:} & \times & \text{Option C:} & \times & \text{Option D:} \\
    3 & & 2 & & 2 & & 3 \\
    \end{array}
    = 36
    
    This includes, for example, a first floor, golf course view apartment with one bedroom and one bathroom.
    This also includes a first floor, lake view apartment with two bedrooms and one bathroom.

11. Multiply the number of choices for each of the three categories:
    \[
    \begin{array}{ccc}
    \text{Gender:} & \times & \text{Age:} & \times & \text{Payment method:} \\
    2 & & 2 & & 2 \\
    \end{array}
    = 8

12. Multiply the number of choices for each of the three groups of highways.
    \[
    \begin{array}{ccc}
    \text{A to B:} & \times & \text{B to C:} & \times & \text{C to D:} \\
    3 & & 2 & & 4 \\
    \end{array}
    = 24

13. Multiply the number of choices for each of the five options:
    \[
    \begin{array}{cccc}
    \text{Color:} & \times & \text{A/C:} & \times & \text{Transmission:} & \times & \text{Windows:} & \times & \text{CD Player:} \\
    6 & & 2 & & 2 & & 2 & & 2 \\
    \end{array}
    = 96

14. Multiply the number of choices for each of the five options:
    \[
    \begin{array}{cccc}
    \text{Color:} & \times & \text{A/C:} & \times & \text{Sun Roof:} & \times & \text{Transmission:} & \times & \text{Brakes:} \\
    9 & & 2 & & 2 & & 2 & & 2 \\
    \end{array}
    = 144

15. Multiply the number of choices for each of the five questions:
    \[
    \begin{array}{cccc}
    \text{Question 1:} & \times & \text{Question 2:} & \times & \text{Question 3:} & \times & \text{Question 4:} & \times & \text{Question 5:} \\
    3 & & 3 & & 3 & & 3 & & 3 \\
    \end{array}
    = 243

16. This situation involves making choices with eight groups of items. Each question is considered a group and each group has 3 choices. Multiply choices: \(3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^8 = 6561\)

17. Multiply the number of choices for each of the three digits:
    \[
    \begin{array}{cc}
    \text{Digit 1:} & \times & \text{Digit 2:} & \times & \text{Digit 3:} \\
    8 & & 2 & & 9 \\
    \end{array}
    = 144
18. Multiply the number of choices for each of the four digits:

Digit 4: Digit 5: Digit 6: Digit 7:
10  ×  10  ×  10  ×  10  =  10,000

19. Multiply the number of choices for each of the letters and digits:

Letter 1: Letter 2: Digit 1: Digit 2: Digit 3:
26  ×  26  ×  10  ×  10  ×  10  =  676,000

20. Multiply the number of choices for each of the four letters:

Letter 1: Letter 2: Letter 3: Letter 4:
2  ×  26  ×  26  ×  26  =  35,152

21. This situation involves making choices with seven groups of items. Each stock is a group, and each group has three choices. Multiply choices: 3 × 3 × 3 × 3 × 3 × 3 × 3 = 3^7 = 2187

22. This situation involves making choices with nine groups of items. Each digit is a group, and each group has ten choices. Multiply choices: 10 × 10 × 10 × 10 × 10 × 10 × 10 × 10 × 10 = 10^9 = 1,000,000,000

26. makes sense

27. makes sense

28. does not make sense; Explanations will vary. Sample explanation: There are 26! or 403,291,461,126,605,635,584,000,000 ways to arrange these letters.

29. makes sense

30. Multiply the number of choices for each of the four digits:

Digit 1: Digit 2: Digit 3: Digit 4:
1-9 0-9 0-9 1,3,5,7,9
9  ×  10  ×  10  ×  5  =  4500

31. Multiply the number of choices for each of the four groups of items:

Bun: Sauce: Lettuce: Tomatoes:
12  ×  30  ×  4  ×  3  =  4320

Total time = 10 × 4320 = 43,200 minutes, which is 43,200 ÷ 60 = 720 hours.

Check Points 11.2

1. There are 5 men to choose from for the first joke. This leaves 5 choices for the second joke. The number of choices then decreases by 1 each time a joke is selected.

1st joke : 2nd joke: 3rd joke: 4th joke: 5th joke: 6th joke:
5  ×  5  ×  4  ×  3  ×  2  ×  1  =  600

2. The number of choices decreases by 1 each time a book is selected.

1st Book: 2nd Book: 3rd Book: 4th Book: 5th Book:
5  ×  4  ×  3  ×  2  ×  1  =  120
3. a. \[
\frac{9!}{6!} = \frac{9 \cdot 8 \cdot 7 \cdot 6!}{6!} = 9 \cdot 8 \cdot 7 = 504
\]

b. \[
16! = \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11!}{11!} = 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 = 524,160
\]

c. \[
\frac{100!}{99!} = \frac{100 \cdot 99!}{99!} = 100
\]

4. a. \[
\frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!} = 7 \cdot 6 \cdot 5 = 840
\]

b. \[
\frac{9!}{(9-5)!} = \frac{9!}{4!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!} = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 15,120
\]

6. There are 7 letters with 2 O's and 3 S's. Thus, \[
\frac{n!}{p!q!} = \frac{7!}{2!3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 420
\]

Concept and Vocabulary Check 11.2

1. factorial; 5; 1; 1

2. \[
\frac{n!}{(n-r)!}
\]

3. \[
\frac{n!}{p!q!}
\]

4. false

5. false

6. true

7. false

Exercise Set 11.2

1. The number of choices decreases by 1 each time a performer is selected.
   \[
   \text{1st Performer: } 6 \times \text{2nd Performer: } 5 \times \text{3rd Performer: } 4 \times \text{4th Performer: } 3 \times \text{5th Performer: } 2 \times \text{6th Performer: } 1 = 720
   \]

2. The number of choices decreases by 1 each time a singer is selected.
   \[
   \text{1st Singer: } 5 \times \text{2nd Singer: } 4 \times \text{3rd Singer: } 3 \times \text{4th Singer: } 2 \times \text{5th Singer: } 1 = 120
   \]
3. The number of choices decreases by 1 each time a sentence is selected.
   \[
   \begin{array}{cccccc}
   1\text{st Sentence} & 2\text{nd Sentence} & 3\text{rd Sentence} & 4\text{th Sentence} & 5\text{th Sentence} \\
   5 & 4 & 3 & 2 & 1 \\
   \end{array}
   \]
   \[= 120\]

4. The number of choices decreases by 1 each time a suspect is selected.
   \[
   \begin{array}{ccccccccc}
   1\text{st Person} & 2\text{nd Person} & 3\text{rd Person} & 4\text{th Person} & 5\text{th Person} & 6\text{th Person} & 7\text{th Person} & 8\text{th Person} \\
   8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\
   \end{array}
   \]
   \[= 40,320\]

5. There is only one choice for the 6th performer. The number of choices decreases by 1 each time a performer is selected.
   \[
   \begin{array}{cccccc}
   1\text{st Performer} & 2\text{nd Performer} & 3\text{rd Performer} & 4\text{th Performer} & 5\text{th Performer} & 6\text{th Performer} \\
   5 & 4 & 3 & 2 & 1 & 1 \\
   \end{array}
   \]
   \[= 120\]

6. There is only one choice for the 5th singer. The number of choices decreases by 1 each time a singer is selected.
   \[
   \begin{array}{cccccc}
   1\text{st Singer} & 2\text{nd Singer} & 3\text{rd Singer} & 4\text{th Singer} & 5\text{th Singer} \\
   4 & 3 & 2 & 1 & 1 \\
   \end{array}
   \]
   \[= 24\]

7. The number of choices decreases by 1 each time a book is selected.
   \[
   \begin{array}{cccccc}
   1\text{st Book} & 2\text{nd Book} & 3\text{rd Book} & 4\text{th Book} & 5\text{th Book} & 6\text{th Book} & 7\text{th Book} & 8\text{th Book} & 9\text{th Book} \\
   9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\
   \end{array}
   \]
   \[= 362,880\]

8. The number of choices decreases by 1 each time a photo is selected.
   \[
   \begin{array}{cccccc}
   1\text{st Photo} & 2\text{nd Photo} & 3\text{rd Photo} & 4\text{th Photo} & 5\text{th Photo} & 6\text{th Photo} & 7\text{th Photo} & 8\text{th Photo} & 9\text{th Photo} & 10\text{th Photo} \\
   10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\
   \end{array}
   \]
   \[= 3,628,800\]

9. There is only one choice each for the first and last sentences. For the other values, the number of choices decreases by 1 each time a sentence is selected.
   \[
   \begin{array}{cccccc}
   1\text{st Sentence} & 2\text{nd Sentence} & 3\text{rd Sentence} & 4\text{th Sentence} & 5\text{th Sentence} \\
   1 & 3 & 2 & 1 & 1 \\
   \end{array}
   \]
   \[= 6\]

10. There is only one choice each for the first and second sentences. For the other values, the number of choices decreases by 1 each time a sentence is selected.
    \[
    \begin{array}{cccccc}
    1\text{st Sentence} & 2\text{nd Sentence} & 3\text{rd Sentence} & 4\text{th Sentence} & 5\text{th Sentence} \\
    1 & 1 & 3 & 2 & 1 \\
    \end{array}
    \]
    \[= 6\]

11. There are two choices for the first movie and one for the second. There is only one choice for the last movie. This leaves two choices for the third movie and one for the fourth.
    \[
    \begin{array}{cccccc}
    1\text{st Movie} & 2\text{nd Movie} & 3\text{rd Movie} & 4\text{th Movie} & 5\text{th Movie} \\
    2 & 1 & 2 & 1 & 1 \\
    \end{array}
    \]
    \[= 4\]

12. There is only one choice each for the fourth and fifth seats. For the other values, the number of choices decreases by 1 each time a seat is selected.
    \[
    \begin{array}{cccccc}
    1\text{st Seat} & 2\text{nd Seat} & 3\text{rd Seat} & 4\text{th Seat} & 5\text{th Seat} & 6\text{th Seat} & 7\text{th Seat} \\
    5 & 4 & 3 & 1 & 1 & 2 & 1 \\
    \end{array}
    \]
    \[= 120\]

13. \[
    \frac{9!}{6!} = \frac{9 \cdot 8 \cdot 7 \cdot 6!}{6!} = 9 \cdot 8 \cdot 7 = 504
    \]
14. \[\frac{12!}{10!} = \frac{12 \cdot 11 \cdot 10!}{10!} = 12 \cdot 11 = 132\]
15. \[\frac{29!}{25!} = \frac{29 \cdot 28 \cdot 27 \cdot 26 \cdot 25!}{25!} = 29 \cdot 28 \cdot 27 \cdot 26 = 570,024\]
16. \[\frac{31!}{28!} = \frac{31 \cdot 30 \cdot 29 \cdot 28!}{28!} = 31 \cdot 30 \cdot 29 = 26,970\]
17. \[\frac{19!}{11!} = \frac{19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11!}{11!} = 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 = 3,047,466,240\]
18. \[\frac{17!}{9!} = \frac{17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9!}{9!} = 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 = 980,179,200\]
19. \[\frac{600!}{599!} \frac{600 \cdot 599!}{599!} = 600\]
20. \[\frac{700!}{699!} \frac{700 \cdot 699!}{699!} = 700\]
21. \[\frac{104!}{102!} \frac{104 \cdot 103 \cdot 102!}{102!} = 104 \cdot 103 = 10,712\]
22. \[\frac{106!}{104!} \frac{106 \cdot 105 \cdot 104!}{104!} = 106 \cdot 105 = 11,130\]
23. \[7! - 3! = 5040 - 6 = 5034\]
24. \[6! - 3! = 720 - 6 = 714\]
25. \[(7 - 3)! = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24\]
26. \[(6 - 3)! = 3! = 3 \cdot 2 \cdot 1 = 6\]
27. \[\binom{12}{4} = 3! = 3 \cdot 2 \cdot 1 = 6\]
28. \[\binom{45}{9} = 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120\]
29. \[\frac{7!}{(7 - 2)!} = \frac{7!}{5!} = \frac{7 \cdot 6 \cdot 5!}{5!} = 7 \cdot 6 = 42\]
30. \[\frac{8!}{(8 - 5)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720\]
31. \[\frac{13!}{(13 - 3)!} = \frac{13!}{10!} \frac{13 \cdot 12 \cdot 11 \cdot 10!}{10!} = 13 \cdot 12 \cdot 11 = 1716\]
32. \[\frac{17!}{(17 - 3)!} = \frac{17!}{14!} \frac{17 \cdot 16 \cdot 15 \cdot 14!}{14!} = 17 \cdot 16 \cdot 15 = 4080\]
33. \[nP_4 = \frac{9!}{(9 - 4)!} \frac{9!}{5!} = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5! = 9 \cdot 8 \cdot 7 \cdot 6 = 3024\]
34. \[nP_5 = \frac{7!}{(7 - 3)!} \frac{7!}{4!} = 7 \cdot 6 \cdot 5 \cdot 4! = 7 \cdot 6 \cdot 5 = 210\]
35. \[nP_5 = \frac{8!}{(8 - 5)!} \frac{8!}{3!} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720\]
36. \[ 10 P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7 = 5040 \]

37. \[ 6 P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 720 \]

38. \[ 9 P_9 = \frac{9!}{(9-9)!} = \frac{9!}{0!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 362,880 \]

39. \[ 8 P_0 = \frac{8!}{(8-0)!} = \frac{8!}{8!} = 1 \]

40. \[ 6 P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 1 \]

41. \[ 10 P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 10 \cdot 9 \cdot 8 = 720 \]

42. \[ 7 P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4 = 840 \]

43. \[ 13 P_7 = \frac{13!}{(13-7)!} = \frac{13!}{6!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6!} = 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 8,648,640 \]

44. \[ 20 P_3 = \frac{20!}{(20-3)!} = \frac{20!}{17!} = \frac{20 \cdot 19 \cdot 18 \cdot 17!}{17!} = 20 \cdot 19 \cdot 18 = 6840 \]

45. \[ 6 P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3!} = 6 \cdot 5 \cdot 4 = 120 \]

46. \[ 8 P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 8 \cdot 7 \cdot 6 = 336 \]

47. \[ 9 P_5 = \frac{9!}{(9-5)!} = \frac{9!}{4!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!} = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 15,120 \]
48. \( P_2 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4 = 840 \)

49. \( \frac{n!}{p!q!} = \frac{6!}{2!2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 180 \)

50. \( \frac{n!}{p!q!} = \frac{7!}{2!2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} = 1260 \)

51. \( \frac{n!}{p!q!r!s!} = \frac{11!}{3!2!2!2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 831,600 \)

52. \( \frac{n!}{p!q!r!} = \frac{9!}{4!2!2!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 3780 \)

53. \( \frac{n!}{p!q!} = \frac{7!}{4!2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 2 \cdot 1} = 105 \)

54. \( \frac{n!}{p!q!r!} = \frac{7!}{2!2!2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 630 \)

55. \( \frac{n!}{p!q!} = \frac{8!}{4!3!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3 \cdot 2 \cdot 1} = 280 \)

56. \( \frac{n!}{p!q!} = \frac{9!}{5!3!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3 \cdot 2 \cdot 1} = 504 \)

63. Because the letter B is repeated in the word BABE, the number of permutations is given by \( \frac{n!}{p!} = \frac{4!}{2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 12 \)

64. makes sense

65. makes sense

66. does not make sense; Explanations will vary. Sample explanation: Since the order does not matter, this situation calls for the combination formula.

67. does not make sense; Explanations will vary. Sample explanation: This situation calls for the formula for permutations of duplicate items.

68. \( i_2 P_{10} = \frac{12!}{(12-10)!} = \frac{12!}{2!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{2!} = 239,500,800 \)

69. Multiply the number of ways to select the two first place horses by the number of orders in which the remaining four horses can finish.

\( P_2 \times C_4 = 15 \times 24 = 360 \)

70. First select 3 out of the 8 jazz groups.

There are \( \frac{8!}{(8-3)!} = \frac{8!}{5!} = 336 \) ways to arrange the 1st, 3rd, and 8th performers.

This leaves 13 groups (5 remaining jazz groups and 8 rock groups) to be arranged.

There are \( \frac{13!}{(13-13)!} = \frac{13!}{0!} = 13! = 6,227,020,800 \) ways to arrange the remaining performers.

The total number of arrangements is found by multiplying these values:

\( 336 \times 6,227,020,800 = 2.09 \times 10^{12} \)

71. There are 5! ways to arrange the women, and 5! ways to arrange the men. The total number of arrangements is found by multiplying these values:

\( (5!) \times (5!) = 120 \times 120 = 14,400 \)
72. Multiply the number of choices for each of the four digits:

\[
\begin{array}{cccc}
\text{Digit 1:} & \text{Digit 2:} & \text{Digit 3:} & \text{Digit 4:} \\
2 & 6 & 6 & 2 \\
\times & \times & \times & \times \\
\end{array}
\]

\[= 144\]

73. \[n \binom{n-2}{r} = \frac{n!}{(n-(n-2))!} = \frac{n!}{2!} = \frac{n(n-1)(n-2) \times \cdots \times 3 \times 2 \times 1}{2} = \frac{n(n-1)(n-2) \times \cdots \times 3}{2} \]

Check Points 11.3

1. a. The order in which you select the DVDs does not matter. This problem involves combinations.

b. Order matters. This problem involves permutations.

2. \[\gamma C_3 = \frac{7!}{(7-3)!3!} = \frac{7!}{4! \cdot 3 \cdot 2 \cdot 1} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35\]

35 such combinations are possible.

3. \[\beta C_4 = \frac{16!}{(16-4)!4!} = \frac{16!}{12! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}{12! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{16 \cdot 15 \cdot 14 \cdot 13}{4 \cdot 3 \cdot 2 \cdot 1} = 1820\]

1820 such hands can be dealt.

4. Choose the male bears: \[\alpha C_2 = \frac{6!}{(6-2)!2!} = \frac{6!}{4! \cdot 2 \cdot 1} = \frac{6 \cdot 5 \cdot 4}{2} = 30\]

Choose the female bears: \[\gamma C_3 = \frac{7!}{(7-3)!3!} = \frac{7!}{4! \cdot 3 \cdot 2 \cdot 1} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = \frac{7 \cdot 6 \cdot 5}{2} = 35\]

Multiply the choices: \[15 \times 35 = 525\]

There are 525 five-bear collections possible.

Concept and Vocabulary Check 11.3

1. \[\frac{n!}{(n-r)!r!}\]

2. \[r!\]

3. false

4. false

Exercise Set 11.3

1. Order does not matter. This problem involves combinations.

2. Order matters. This problem involves permutations.

3. Order matters. This problem involves permutations.
4. Order does not matter. This problem involves combinations.

5. \( C_5 = \frac{6!}{(6-5)!5!} = \frac{6!}{1!5!} = \frac{6 \cdot 5!}{1 \cdot 5!} = 6 \)

6. \( C_7 = \frac{8!}{(8-7)!7!} = \frac{8!}{1!7!} = \frac{8 \cdot 7!}{1 \cdot 7!} = 8 \)

7. \( C_8 = \frac{9!}{(9-5)!5!} = \frac{9!}{4!5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 5!} = 126 \)

8. \( C_6 = \frac{10!}{(10-6)!6!} = \frac{10!}{4!6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 6!} = 210 \)

9. \( C_4 = \frac{11!}{(11-4)!4!} = \frac{11!}{7!4!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 330 \)

10. \( C_5 = \frac{12!}{(12-5)!5!} = \frac{12!}{7!5!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 792 \)

11. \( C_1 = \frac{8!}{(8-1)!1!} = \frac{8!}{7!1!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 8 \)

12. \( C_1 = \frac{7!}{(7-1)!1!} = \frac{7!}{6!1!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 7 \)

13. \( C_1 = \frac{7!}{(7-7)!7!} = \frac{7!}{0!7!} = 1 \)

14. \( C_4 = \frac{4!}{(4-4)!4!} = \frac{4!}{0!4!} = 1 \)

15. \( C_3 = \frac{30!}{(30-3)!3!} = \frac{30!}{27!3!} = \frac{30 \cdot 29 \cdot 28 \cdot 27!}{27! \cdot 3 \cdot 2 \cdot 1} = 4060 \)

16. \( C_4 = \frac{25!}{(25-4)!4!} = \frac{25!}{21!4!} = \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21!}{21! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 12,650 \)

17. \( C_0 = \frac{5!}{(5-0)!0!} = \frac{5!}{5!0!} = 1 \)

18. \( C_0 = \frac{6!}{(6-0)!0!} = \frac{6!}{6!0!} = 1 \)

19. \( C_3 = \frac{7!}{(3-3)!3!} = \frac{7!}{3!3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{3 \cdot 2 \cdot 1} = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{3 \cdot 2 \cdot 1} = \frac{35 \cdot 5}{5} = 7 \)
20. \( \frac{10!}{10!} \frac{20!}{20!} \frac{6!}{6!} \frac{10!}{10!} \frac{8!}{8!} \frac{7!}{7!} \frac{3!}{3!} \frac{10!}{10!} \frac{9!}{9!} \frac{8!}{8!} \frac{7!}{7!} \frac{3!}{3!} \frac{2!}{2!} \frac{1!}{1!} = \frac{60}{15} = 8 \)

21. \( \frac{7!}{3!} \frac{7!}{7!-3!} \frac{4!}{3!} \frac{7!}{7!} \frac{7!}{7!} \frac{7!}{7!} \frac{7!}{7!} = 0 \)

22. \( \frac{20!}{2!} \frac{20!}{20!} \frac{6!}{6!} \frac{20!}{20!} \frac{20!}{20!} \frac{6!}{6!} \frac{20!}{20!} \frac{6!}{6!} \frac{20!}{20!} \frac{6!}{6!} = 0 \)

23. \( \frac{3!}{3!} \frac{4!}{3!} \frac{1}{4!} = \frac{1}{4} \)

24. \( \frac{5!}{(5-3)!} \frac{2!}{2!} \frac{6!}{6!} \frac{10!}{10!} \frac{2!}{2!} \frac{5!}{5!} \frac{4!}{4!} = 83 \)

25. \( \frac{98!}{96!} \frac{7!}{(5-4)!} \frac{5!}{5!} \frac{97!}{96!} = \frac{7!}{5!} \frac{4!}{4!} \frac{7!}{7!} = 35 \)

26. \( \frac{10!}{6!} \frac{46!}{44!} \frac{10!}{10!} \frac{46!}{44!} = \frac{10!}{6!} \frac{46!}{44!} \frac{2!}{2!} = \frac{10!}{6!} \frac{46!}{44!} \frac{2!}{2!} = 2070 \)

27. \( \frac{4!}{(4-2)!} \frac{6!}{6!} \frac{18!}{18!} \frac{18!}{18!} = \frac{4!}{(4-2)!} \frac{6!}{6!} \frac{18!}{18!} \frac{18!}{18!} = 36 \)

28. \( \frac{5!}{(5-1)!} \frac{7!}{7!} \frac{5!}{5!} \frac{5!}{5!} = \frac{5!}{(5-1)!} \frac{7!}{7!} \frac{5!}{5!} \frac{5!}{5!} = 2160 \)

29. \( \frac{6!}{(6-3)!} \frac{6!}{6!} \frac{6!}{6!} \frac{3!}{3!} = 20 \)

30. \( \frac{11!}{11-4!} \frac{11!}{11!} = \frac{11!}{11!} \frac{11!}{11!} \frac{9!}{9!} \frac{3!}{3!} = 330 \)
31. \[ \binom{12}{4} = \frac{12!}{(12-4)!4!} = \frac{12!}{8!4!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8! \cdot 3 \cdot 2 \cdot 1} = 495 \]

32. \[ \binom{14}{6} = \frac{14!}{(14-6)!6!} = \frac{14!}{8!6!} = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8! \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 3003 \]

33. \[ \binom{17}{8} = \frac{17!}{(17-8)!8!} = \frac{17!}{9!8!} = \frac{17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{9! \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 24,310 \]

34. \[ \binom{100}{18} = \frac{100!}{(100-18)!18!} = \frac{100!}{82!18!} \]

35. \[ \binom{53}{6} = \frac{53!}{(53-6)!6!} = \frac{53!}{47!6!} = \frac{53 \cdot 52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47!}{47! \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 22,957,480 \]

36. \[ \binom{59}{6} = \frac{59!}{(59-6)!6!} = \frac{59!}{53!6!} = \frac{59 \cdot 58 \cdot 57 \cdot 56 \cdot 55 \cdot 54 \cdot 53!}{53! \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 45,057,474 \]

37. Choose the men: \[ \binom{7}{4} = \frac{7!}{(7-4)!4!} = \frac{7!}{3!4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3 \cdot 2 \cdot 1} = 35 \]
Choose the women: \[ \binom{7}{5} = \frac{7!}{(7-5)!5!} = \frac{7!}{2!5!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{2 \cdot 1} = 21 \]
Multiply the choices: \[ 35 \cdot 21 = 735 \]

38. Choose the professors: \[ \binom{5}{2} = \frac{5!}{(5-2)!2!} = \frac{5!}{3!2!} = \frac{5 \cdot 4 \cdot 3!}{3 \cdot 2 \cdot 1} = 10 \]
Choose the students: \[ \binom{15}{10} = \frac{15!}{(15-10)!10!} = \frac{15!}{5!10!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 10!} = 3003 \]
Multiply the choices: \[ 10 \cdot 3003 = 30,030 \]

39. Choose the Republicans: \[ \binom{55}{4} = \frac{55!}{(55-4)!4!} = \frac{55!}{51!4!} = \frac{55 \cdot 54 \cdot 53 \cdot 52 \cdot 51!}{51! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 341,055 \]
Choose the Democrats: \[ \binom{44}{3} = \frac{44!}{(44-3)!3!} = \frac{44!}{41!3!} = \frac{44 \cdot 43 \cdot 42 \cdot 41!}{3 \cdot 2 \cdot 1 \cdot 41!} = 13,244 \]
Multiply the choices: \[ 341,055 \cdot 13,244 = 4,516,932,420 \]

40. Choose the multiple-choice questions: \[ \binom{10}{8} = \frac{10!}{(10-8)!8!} = \frac{10!}{2!8!} = \frac{10 \cdot 9 \cdot 8!}{2 \cdot 1 \cdot 8!} = 45 \]
Choose the open-ended problems: \[ \binom{5}{3} = \frac{5!}{(5-3)!3!} = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3!}{2 \cdot 1 \cdot 3!} = 10 \]
Multiply the choices: \[ 45 \cdot 10 = 450 \]

41. \[ \binom{6}{2} = \frac{6!}{2!} = 6 \cdot 5 \cdot 4 \cdot 3 = 360 \text{ ways} \]

42. \[ \binom{40}{8} = \frac{40!}{32!8!} = 76,904,685 \text{ selections} \]
43. \( _{13}C_6 = \frac{13!}{7! \cdot 6!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 1716 \text{ ways} \)

44. \( _{50}P_3 = \frac{50!}{47!} = 50 \cdot 49 \cdot 48 = 177,600 \text{ ways} \)

45. \( _{20}C_3 = \frac{20!}{17! \cdot 3!} = \frac{20 \cdot 19 \cdot 18}{3 \cdot 2 \cdot 1} = 1140 \text{ ways} \)

46. \( _{50}C_3 = \frac{50!}{47! \cdot 3!} = \frac{50 \cdot 49 \cdot 48}{3 \cdot 2 \cdot 1} = 19,600 \text{ ways} \)

47. \( _7P_4 = \frac{7!}{3!} = 840 \text{ passwords} \)

48. \( _9P_3 = \frac{9!}{4!} = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 15,120 \text{ ways} \)

49. \( _{15}P_3 = \frac{15!}{12!} = 15 \cdot 14 \cdot 13 = 2730 \text{ cones} \)

50. \( _{31}C_3 = \frac{31!}{28! \cdot 3!} = \frac{31 \cdot 30 \cdot 29}{3 \cdot 2 \cdot 1} = 4495 \text{ bowls} \)

51. \( _5C_2 = \frac{5!}{3! \cdot 2!} = \frac{5 \cdot 4}{2 \cdot 1} = 10 \text{ outcomes} \)

52. \( _6C_3 = \frac{6!}{3! \cdot 3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20 \text{ groups} \)

53. \( 3 \times 2 \times 2 = 12 \text{ outcomes} \)

54. \( _4C_2 = \frac{4!}{2! \cdot 2!} = \frac{4 \cdot 3}{2 \cdot 1} = 6 \text{ outcomes} \)

55. \( _5C_3 = \frac{5!}{2! \cdot 3!} = \frac{5 \cdot 4}{2 \cdot 1} = 10 \text{ outcomes} \)

56. \( _4C_3 = \frac{4!}{1! \cdot 3!} = \frac{4}{1} = 4 \text{ ways} \)

57. Choose the Democrats: \( _4C_2 = \frac{4!}{2! \cdot 2!} = \frac{4 \cdot 3}{2 \cdot 1} = 6 \)

Choose the Republicans: \( _5C_2 = \frac{5!}{3! \cdot 2!} = \frac{5 \cdot 4}{2 \cdot 1} = 10 \)

Multiply the choices: \( 6 \cdot 10 = 60 \)

58. \( _6C_2 = \frac{6!}{4! \cdot 2!} = \frac{6 \cdot 5}{2 \cdot 1} = 15 \text{ ways} \)

59. \( _{12}P_5 = \frac{12!}{(12-5)!} = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 = 95,040 \text{ ways} \)
60. \[ P_2 = \frac{7!}{(7-2)!} = \frac{7!}{5!} = 7 \cdot 6 = 42 \] outcomes

61. \[ P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720 \text{ ways} \]

62. \[ P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \text{ ways} \]

63. \[ C_3 = \frac{6!}{(6-3)!3!} = \frac{6!}{3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20 \text{ ways} \]

64. \[ C_3 = \frac{6!}{(6-2)!2!} = \frac{6!}{4!2!} = \frac{6 \cdot 5 \cdot 4}{2 \cdot 1 \cdot 4!} = 15 \text{ ways} \]

65. \[ P_2 = \frac{4!}{(4-4)!} = 4 \text{ ways} \]

66. \[ 2 \cdot P_3 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 2 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 240 \text{ ways} \]

67. \[ 2 \cdot C_2 = 2 \cdot \frac{4!}{(4-2)!2!} = 2 \cdot \frac{4!}{2!} = 2 \cdot \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 12 \text{ ways} \]

68. \[ 2 \cdot C_3 = 2 \cdot \frac{4!}{(4-3)!3!} = 2 \cdot \frac{4!}{1!} = 2 \cdot \frac{4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 3!} = 8 \text{ ways} \]

72. does not make sense; Explanations will vary. Sample explanation: Since order matters, the permutation formula is necessary.

73. does not make sense; Explanations will vary. Sample explanation: Since order matters, the permutation formula is necessary.

74. makes sense

75. makes sense

77. Selections for 6/53 lottery:
   \[ _{53}C_6 = \frac{53!}{(53-6)!6!} = \frac{53!}{47!6!} = \frac{53 \cdot 52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47!}{47!6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 22,957,480 \]

   Selections for 5/36 lottery:
   \[ _{36}C_5 = \frac{36!}{(36-5)!5!} = \frac{36!}{31!5!} = \frac{36 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31!}{31!5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 376,992 \]

   The 5/36 lottery is easier to win because there are fewer possible selections.
Chapter 11  Counting Methods and Probability Theory

78. \[ nPr = 6 \cdot nCr \]
\[
\frac{n!}{(n-r)!} = 6 \cdot \frac{n!}{(n-r)!r!} \]  \[\text{[apply the cross products principle]}\]
\[
n!(n-r)!r! = 6 \cdot n!(n-r)! \]  \[\text{[divide both sides by } n!(n-r)!]\]
\[
\frac{n!(n-r)!r!}{n!(n-r)!} = \frac{6 \cdot n!(n-r)!}{n!(n-r)!} \]
\[
r! = 6 \]
\[
r! = 3 \cdot 2 \cdot 1 \]
\[
r = 3 \]

No, there is not enough information to determine the value of \( n \). The number of permutations will be six times the number of combinations for all values of \( n \) when \( r = 3 \).

79. For a group of 20 people:
\[
\begin{align*}
20 C_2 &= \frac{20!}{(20-2)!2!} = \frac{20!}{18!2!} = \frac{20 \cdot 19 \cdot 18!}{18!2 \cdot 1} = 190 \text{ handshakes} \\
\text{Time} &= 3 \times 190 = 570 \text{ seconds, which gives } 570 + 60 = 9.5 \text{ minutes.} 
\end{align*}
\]
For a group of 40 people:
\[
\begin{align*}
40 C_2 &= \frac{40!}{(40-2)!2!} = \frac{40!}{38!2!} = \frac{40 \cdot 39 \cdot 38!}{38!2 \cdot 1} = 780 \text{ handshakes} \\
\text{Time} &= 3 \times 780 = 2340 \text{ seconds, which gives } 2340 + 60 = 39 \text{ minutes.} 
\end{align*}
\]

80. Since there are 5 defective phones, we must select 4 out of the 15 good phones.
\[
15 C_4 = \frac{15!}{(15-4)!4!} = \frac{15!}{11!4!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11!}{11!4 \cdot 3 \cdot 2 \cdot 1} = 1365 
\]

Check Points 11.4

1. a. The event of getting a 2 can occur in one way.
\[
P(2) = \frac{\text{number of ways a 2 can occur}}{\text{total number of possible outcomes}} = \frac{1}{6} 
\]

b. The event of getting a number less than 4 can occur in three ways: 1, 2, 3.
\[
P(\text{less than 4}) = \frac{\text{number of ways a number less than 4 can occur}}{\text{total number of possible outcomes}} = \frac{3}{6} = \frac{1}{2} 
\]

c. The event of getting a number greater than 7 cannot occur.
\[
P(\text{greater than 7}) = \frac{\text{number of ways a number greater than 7 can occur}}{\text{total number of possible outcomes}} = \frac{0}{6} = 0 
\]
The probability of an event that cannot occur is 0.

d. The event of getting a number less than 7 can occur in six ways: 1, 2, 3, 4, 5, 6.
\[
P(\text{less than 7}) = \frac{\text{number of ways a number less than 7 can occur}}{\text{total number of possible outcomes}} = \frac{6}{6} = 1 
\]
The probability of any certain event is 1.
2. a. \[ P(\text{ace}) = \frac{\text{number of ways a ace can occur}}{\text{total number of possibilities}} = \frac{4}{52} = \frac{1}{13} \]

b. \[ P(\text{red card}) = \frac{\text{number of ways a red card can occur}}{\text{total number of possible outcomes}} = \frac{26}{52} = \frac{1}{2} \]

c. \[ P(\text{red king}) = \frac{\text{number of ways a red king can occur}}{\text{total number of possible outcomes}} = \frac{2}{52} = \frac{1}{26} \]

3. The table shows the four equally likely outcomes. The \( Cc \) and \( cC \) children will be carriers who are not actually sick.
\[ P(\text{carrier, not sick}) = P(Cc) = \frac{\text{number of ways } Cc \text{ or } cC \text{ can occur}}{\text{total number of possible outcomes}} = \frac{2}{4} = \frac{1}{2} \]

4. a. \[ P(\text{never married}) = \frac{\text{number of persons never married}}{\text{total number of U.S. adults}} = \frac{74}{242} = 0.31 \]

b. \[ P(\text{male}) = \frac{\text{number of males}}{\text{total number of U.S. adults}} = \frac{118}{242} = 0.49 \]

Concept and Vocabulary Check 11.4
1. sample space
2. \( P(E) \); number of outcomes in \( E \); total number of possible outcomes
3. 52; hearts; diamonds; clubs; spades
4. empirical
5. true
6. false
7. true
8. false

Exercise Set 11.4
1. \[ P(4) = \frac{\text{number of ways a 4 can occur}}{\text{total number of possible outcomes}} = \frac{1}{6} \]

2. \[ P(5) = \frac{\text{number of ways a five can occur}}{\text{total number of possible outcomes}} = \frac{1}{6} \]

3. \[ P(\text{odd number}) = \frac{\text{number of ways an odd number can occur}}{\text{total number of possible outcomes}} = \frac{3}{6} = \frac{1}{2} \]
4. \( P(\text{greater than 3}) = \frac{\text{number of ways a number greater than 3 can occur}}{\text{total number of possible outcomes}} = \frac{3}{6} = \frac{1}{2} \)

5. \( P(\text{less than 3}) = \frac{\text{number of ways a number less than 3 can occur}}{\text{total number of possible outcomes}} = \frac{2}{6} = \frac{1}{3} \)

6. \( P(\text{greater than 4}) = \frac{\text{number of ways a number greater than 4 can occur}}{\text{total number of possible outcomes}} = \frac{2}{6} = \frac{1}{3} \)

7. \( P(\text{less than 20}) = \frac{\text{number of ways a number less than 20 can occur}}{\text{total number of possible outcomes}} = \frac{6}{6} = 1 \)

8. \( P(\text{less than 8}) = \frac{\text{number of ways a number less than 8 can occur}}{\text{total number of possible outcomes}} = \frac{6}{6} = 1 \)

9. \( P(\text{greater than 20}) = \frac{\text{number of ways a number greater than 20 can occur}}{\text{total number of possible outcomes}} = \frac{0}{6} = 0 \)

10. \( P(\text{greater than 8}) = \frac{\text{number of ways a number greater than 8 can occur}}{\text{total number of possible outcomes}} = \frac{0}{6} = 0 \)

11. \( P(\text{queen}) = \frac{\text{number of ways a queen can occur}}{\text{total number of possibilities}} = \frac{4}{52} = \frac{1}{13} \)

12. \( P(\text{jack}) = \frac{\text{number of ways a jack can occur}}{\text{total number of possibilities}} = \frac{4}{52} = \frac{1}{13} \)

13. \( P(\text{club}) = \frac{\text{number of ways a club can occur}}{\text{total number of possibilities}} = \frac{13}{52} = \frac{1}{4} \)

14. \( P(\text{diamond}) = \frac{\text{number of ways a diamond can occur}}{\text{total number of possibilities}} = \frac{13}{52} = \frac{1}{4} \)

15. \( P(\text{picture card}) = \frac{\text{number of ways a picture card can occur}}{\text{total number of possibilities}} = \frac{12}{52} = \frac{3}{13} \)

16. \( P(\text{greater than 3 and less than 7}) = \frac{\text{number of ways a card greater than 3 and less than 7 can occur}}{\text{total number of possibilities}} = \frac{12}{52} = \frac{3}{13} \)

17. \( P(\text{queen of spades}) = \frac{\text{number of ways a queen of spades can occur}}{\text{total number of possibilities}} = \frac{1}{52} \)

18. \( P(\text{ace of clubs}) = \frac{\text{number of ways an ace of clubs can occur}}{\text{total number of possibilities}} = \frac{1}{52} \)

19. \( P(\text{diamond and spade}) = \frac{\text{number of ways a diamond and a spade can occur}}{\text{total number of possibilities}} = \frac{0}{52} = 0 \)
20. \[ P(\text{green heart}) = \frac{\text{number of ways a card with a green heart can occur}}{\text{total number of possibilities}} = \frac{0}{52} = 0 \]

21. \[ P(\text{two heads}) = \frac{\text{number of ways two heads can occur}}{\text{total number of possibilities}} = \frac{1}{4} \]

22. \[ P(\text{two tails}) = \frac{\text{number of ways two tails can occur}}{\text{total number of possibilities}} = \frac{1}{4} \]

23. \[ P(\text{same on each toss}) = \frac{\text{number of ways the same outcome on each toss can occur}}{\text{total number of possibilities}} = \frac{1}{4} \]

24. \[ P(\text{different on each toss}) = \frac{\text{number of ways different outcomes on each toss can occur}}{\text{total number of possibilities}} = \frac{1}{4} \]

25. \[ P(\text{head on second toss}) = \frac{\text{number of ways a head on the second toss can occur}}{\text{total number of possibilities}} = \frac{1}{2} \]

26. \[ P(\text{at least one head}) = \frac{\text{number of ways at least one head can occur (HH, HT, TH)}}{\text{total number of possibilities}} = \frac{3}{4} \]

27. \[ P(\text{exactly one female child}) = \frac{\text{number of ways exactly one female child can occur}}{\text{total number of possibilities}} = \frac{3}{8} \]

28. \[ P(\text{exactly one male child}) = \frac{\text{number of ways exactly one male child can occur}}{\text{total number of possibilities}} = \frac{3}{8} \]

29. \[ P(\text{exactly two male children}) = \frac{\text{number of ways exactly two male children can occur}}{\text{total number of possibilities}} = \frac{3}{8} \]

30. \[ P(\text{exactly two female children}) = \frac{\text{number of ways exactly two female children can occur}}{\text{total number of possibilities}} = \frac{3}{8} \]

31. \[ P(\text{at least one male child}) = \frac{\text{number of ways at least one male child can occur}}{\text{total number of possibilities}} = \frac{7}{8} \]

32. \[ P(\text{at least two female children}) = \frac{\text{number of ways at least two female children can occur}}{\text{total number of possibilities}} = \frac{4}{8} = \frac{1}{2} \]

33. \[ P(\text{four male children}) = \frac{\text{number of ways four male children can occur}}{\text{total number of possibilities}} = \frac{0}{8} = 0 \]

34. \[ P(\text{fewer than four female children}) = \frac{\text{number of ways fewer than four female children can occur}}{\text{total number of possibilities}} = \frac{8}{8} = 1 \]

35. \[ P(\text{two even numbers}) = \frac{\text{number of ways two even numbers can occur}}{\text{total number of possibilities}} = \frac{9}{36} = \frac{1}{4} \]
Chapter 11  Counting Methods and Probability Theory

36. \[ P(\text{two odd numbers}) = \frac{\text{number of ways two odd numbers can occur}}{\text{total number of possibilities}} = \frac{9}{36} = \frac{1}{4} \]

37. \[ P(\text{two numbers whose sum is 5}) = \frac{\text{number of ways two numbers whose sum is 5 can occur}}{\text{total number of possibilities}} = \frac{4}{36} = \frac{1}{9} \]

38. \[ P(\text{two numbers of whose sum is 6}) = \frac{\text{number of ways two numbers whose sum is 6 can occur}}{\text{total number of possibilities}} = \frac{5}{36} \]

39. \[ P(\text{two numbers whose sum exceeds 12}) = \frac{\text{number of ways two numbers whose sum exceeds 12 can occur}}{\text{total number of possibilities}} = \frac{0}{36} = 0 \]

40. \[ P(\text{two numbers whose sum is less than 13}) = \frac{\text{number of ways two numbers whose sum is less than 13 can occur}}{\text{total number of possibilities}} = \frac{36}{36} = 1 \]

41. \[ P(\text{red region}) = \frac{\text{number of ways a red region can occur}}{\text{total number of possibilities}} = \frac{3}{10} \]

42. \[ P(\text{yellow region}) = \frac{\text{number of ways a yellow region can occur}}{\text{total number of possibilities}} = \frac{2}{10} = \frac{1}{5} \]

43. \[ P(\text{blue region}) = \frac{\text{number of ways a blue region can occur}}{\text{total number of possibilities}} = \frac{2}{10} = \frac{1}{5} \]

44. \[ P(\text{brown region}) = \frac{\text{number of ways a brown region can occur}}{\text{total number of possibilities}} = \frac{3}{10} \]

45. \[ P(\text{region that is red or blue}) = \frac{\text{number of ways a region that is red or blue can occur}}{\text{total number of possibilities}} = \frac{5}{10} = \frac{1}{2} \]

46. \[ P(\text{region that is yellow or brown}) = \frac{\text{number of ways a region that is yellow or brown can occur}}{\text{total number of possibilities}} = \frac{5}{10} = \frac{1}{2} \]

47. \[ P(\text{region that is red and blue}) = \frac{\text{number of ways a region that is red and blue can occur}}{\text{total number of possibilities}} = \frac{0}{10} = 0 \]

48. \[ P(\text{region that is yellow and brown}) = \frac{\text{number of ways a region that is yellow and brown can occur}}{\text{total number of possibilities}} = \frac{0}{10} = 0 \]

49. \[ P(\text{sickle cell anemia}) = \frac{\text{number of ways sickle cell anemia can occur}}{\text{total number of possibilities}} = \frac{1}{4} \]

50. \[ P(\text{sickle cell trait}) = \frac{\text{number of ways sickle cell trait can occur}}{\text{total number of possibilities}} = \frac{2}{4} = \frac{1}{2} \]

51. \[ P(\text{healthy}) = \frac{\text{number of ways a healthy child can occur}}{\text{total number of possibilities}} = \frac{1}{4} \]
52. \( P(\text{sickle cell anemia}) = \frac{\text{number of ways sickle cell anemia can occur}}{\text{total number of possibilities}} = \frac{0}{4} = 0 \)

53. \( P(\text{sickle cell trait}) = \frac{\text{number of ways sickle cell trait can occur}}{\text{total number of possibilities}} = \frac{2}{4} = \frac{1}{2} \)

54. \( P(\text{healthy}) = \frac{\text{number of ways a healthy child can occur}}{\text{total number of possibilities}} = \frac{2}{4} = \frac{1}{2} \)

55. \( P(\text{male}) = \frac{\text{number of males}}{\text{total number of Americans living alone}} = \frac{12.5}{29.3} = 0.43 \)

56. \( P(\text{female}) = \frac{\text{number of females}}{\text{total number of Americans living alone}} = \frac{16.8}{29.3} = 0.57 \)

57. \( P(25–34 \text{ age range}) = \frac{\text{number in 25–34 age range}}{\text{total number of Americans living alone}} = \frac{3.8}{29.3} = 0.13 \)

58. \( P(35–44 \text{ age range}) = \frac{\text{number in 35–44 age range}}{\text{total number of Americans living alone}} = \frac{4.2}{29.3} = 0.14 \)

59. \( P(\text{woman in 15–24 age range}) = \frac{\text{number of women in 15–24 age range}}{\text{total number of Americans living alone}} = \frac{0.8}{29.3} = 0.03 \)

60. \( P(\text{man in 45–64 age range}) = \frac{\text{number of men in 45–64 age range}}{\text{total number of Americans living alone}} = \frac{4.3}{29.3} = 0.15 \)

<table>
<thead>
<tr>
<th>Table For #61–66</th>
<th>Moved to Same State</th>
<th>Moved to Different State</th>
<th>Moved to Different Country</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Owner</td>
<td>11.7</td>
<td>2.8</td>
<td>0.3</td>
<td>14.8</td>
</tr>
<tr>
<td>Renter</td>
<td>18.7</td>
<td>4.5</td>
<td>1.0</td>
<td>24.2</td>
</tr>
<tr>
<td>Total</td>
<td>30.4</td>
<td>7.3</td>
<td>1.3</td>
<td>39.0</td>
</tr>
</tbody>
</table>

61. \( P(\text{owner}) = \frac{\text{number of owners}}{\text{total number of Americans who moved in 2004}} = \frac{14.8}{39.0} = 0.38 \)

62. \( P(\text{renter}) = \frac{\text{number of renters}}{\text{total number of Americans who moved in 2004}} = \frac{24.2}{39.0} = 0.62 \)

63. \( P(\text{moved within state}) = \frac{\text{number that moved within state}}{\text{total number of Americans who moved in 2004}} = \frac{30.4}{39.0} = 0.78 \)

64. \( P(\text{moved to different country}) = \frac{\text{number that moved to different country}}{\text{total number of Americans who moved in 2004}} = \frac{1.3}{39.0} = 0.03 \)

65. \( P(\text{renter who moved to different state}) = \frac{\text{number of renters who moved to a different state}}{\text{total number of Americans who moved in 2004}} = \frac{4.5}{39.0} = 0.12 \)

Copyright © 2015 Pearson Education, Inc.
66. \( P(\text{owner who moved to different state}) = \frac{\text{number of owners who moved to a different state}}{\text{total number of Americans who moved in 2004}} = \frac{2.8}{39.0} = 0.07 \)

<table>
<thead>
<tr>
<th>Table For #67–70</th>
<th>Less Than 4 Years High School</th>
<th>4 Years High School Only</th>
<th>Some College (Less than 4 years)</th>
<th>4 Years College (or More)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>3.6</td>
<td>5.0</td>
<td>2.6</td>
<td>3.9</td>
<td>15.1</td>
</tr>
<tr>
<td>Female</td>
<td>5.2</td>
<td>8.0</td>
<td>4.1</td>
<td>3.0</td>
<td>20.3</td>
</tr>
<tr>
<td>Total</td>
<td>8.8</td>
<td>13.0</td>
<td>6.7</td>
<td>6.9</td>
<td>35.4</td>
</tr>
</tbody>
</table>

67. \( P(\text{had less than 4 years of high school}) = \frac{\text{number with less than 4 years of high school}}{\text{total number of Americans aged 25 and older}} = \frac{29}{174} = \frac{1}{6} \)

68. \( P(\text{had 4 years of high school only}) = \frac{\text{number with 4 years of high school only}}{\text{total number of Americans aged 25 and older}} = \frac{56}{174} = \frac{28}{87} \)

69. \( P(\text{a woman with 4 years of college or more}) = \frac{\text{number of women with 4 years of college or more}}{\text{total number of Americans aged 25 and older}} = \frac{22}{174} = \frac{11}{87} \)

70. \( P(\text{a man with 4 years of college or more}) = \frac{\text{number of men with 4 years of college or more}}{\text{total number of Americans aged 25 and older}} = \frac{23}{174} \)

79. does not make sense; Explanations will vary. Sample explanation: Even if there are only two choices, it does not necessarily follow that they are equally likely to be selected.

80. does not make sense; Explanations will vary. Sample explanation: The probability cannot be greater than 1 (100%).

81. makes sense

82. makes sense

83. The area of the target is \((12 \text{ in.})^2 = 144 \text{ in.}^2\)

The area of the yellow region is \((9 \text{ in.})^2 - (6 \text{ in.})^2 + (3 \text{ in.})^2 = 54 \text{ in.}^2\)

The probability that the dart hits a yellow region is \(\frac{54 \text{ in.}^2}{144 \text{ in.}^2} = 0.375\)

84. First count the number of three-digit numbers that read the same forward and backward:

\[
\begin{align*}
\text{Digit 1:} & \quad 1 - 9 \\
\text{Digit 2:} & \quad 0 - 9 \\
\text{Digit 3:} & \quad \text{Same as 1st digit} \\
9 \times 10 \times 1 & = 90 \\
\end{align*}
\]

\(P(\text{three-digit number reads the same forward and backward}) = \frac{\text{number of three-digit numbers that read the same forward and backward}}{\text{total number of three-digit numbers}} = \frac{90}{900} = \frac{1}{10} \)
Check Points 11.5

1. total number of permutations = \(6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720\)

   For the given outcome there is 1 choice (first name beginning with G) for the first joke, which would leave 4 choices for the last joke (the 4 remaining men). The remaining jokes have 4, 3, 2, and 1 choice respectively.

   \[
   \begin{array}{cccccc}
   \text{1st:} & \text{2nd:} & \text{3rd:} & \text{4th:} & \text{5th:} & \text{6th:} \\
   1 & \times & 4 & \times & 3 & \times 2 & \times 1 & \times 4 \\
   \end{array}
   \]

   \[P(\text{first joke is by a man whose name begins with G and the last is by a man}) = \frac{96}{720} = \frac{2}{15}\]

2. Total number of Powerball selections: \(\binom{59}{5} \cdot \binom{35}{1}\)

   \[
   \begin{align*}
   \binom{59}{5} &= \frac{59!}{(59-5)!5!} \\
   &= \frac{59!}{54!5!} \\
   &= \frac{59 \cdot 58 \cdot 57 \cdot 56 \cdot 55}{54 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\
   &= \frac{5,006,386}{35} \\
   &= 175,223,510
   \end{align*}
   \]

   Number of selections that match 4 out of 5 white balls and the Powerball:

   \[
   \binom{5}{4} \cdot \binom{54}{1} \cdot 1 = 270
   \]

   \[P(\text{matching 4 of the 5 white balls and the powerball}) = \frac{270}{175,223,510} = \frac{27}{17,522,351} = 0.000001541 = 1.541 \times 10^{-6}\]

3. total number of combinations: \(\binom{10}{3} = \frac{10!}{(10-3)!3!} = \frac{10!}{7!3!} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{7! \cdot 2 \cdot 1} = 120\)

   a. total number of combinations of 3 men: \(\binom{6}{3} = \frac{6!}{(6-3)!3!} = \frac{6!}{3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 20\)

   \[P(\text{3 men}) = \frac{\text{number of combinations with 3 men}}{\text{total number of combinations}} = \frac{20}{120} = \frac{1}{6}\]

   b. Select 2 out of 6 men: \(\binom{6}{2} = \frac{6!}{(6-2)!2!} = \frac{6!}{4!2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 15\)

   Select 1 out of 4 women: \(\binom{4}{1} = \frac{4!}{(4-1)!1!} = \frac{4!}{3!1!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 1} = 4\)

   total number of combinations of 2 men and 1 woman: \(15 \times 4 = 60\)

   \[P(\text{2 men, 1 woman}) = \frac{\text{number of combinations with 2 men, 1 woman}}{\text{total number of combinations}} = \frac{60}{120} = \frac{1}{2}\]
Concept and Vocabulary Check 11.5

1. permutations; the total number of possible permutations
2. 1; combinations
3. true
4. false

Exercise Set 11.5

1. a. \(5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120\)
   
   b. \(\begin{align*}
   &\text{Martha} &\text{Lee, Nancy, Paul} &\text{Armando} \\
   &1 \times &3 \times &2 \times &1 \times &1 = 6
   \end{align*}\)

   c. \(P(\text{Martha first and Armando last}) = \frac{6}{120} = \frac{1}{20}\)

2. a. \(6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720\)

   b. \(\begin{align*}
   &\text{1st woman: } \text{1st man: } \text{2nd woman: } \text{2nd man: } \text{3rd woman: } \text{3rd man:} \\
   &3 \times 3 \times 2 \times 2 \times 1 \times 1 = 36
   \end{align*}\)

   c. \(P(\text{first person is a woman and line alternates by gender}) = \frac{36}{720} = \frac{1}{20}\)

3. a. total number of permutations = \(6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720\)
   number of permutations with E first = \(1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120\)

   \(P(\text{E first}) = \frac{\text{number of permutations with E first}}{\text{total number of permutations}} = \frac{120}{720} = \frac{1}{6}\)

   b. number of permutations with C fifth and B last = \(4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 24\)

   \(P(\text{C fifth and B last}) = \frac{\text{number of permutations with C fifth and B last}}{\text{total number of permutations}} = \frac{24}{720} = \frac{1}{30}\)

   c. \(P(\text{D, E, C, A, B, F}) = \frac{\text{number of permutations with order D, E, C, A, B, F}}{\text{total number of permutations}} = \frac{1}{720}\)

4. a. total number of permutations = \(7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040\)
   number of permutations with D first = \(1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720\)

   \(P(\text{D first}) = \frac{\text{number of permutations with D first}}{\text{total number of permutations}} = \frac{720}{5040} = \frac{1}{7}\)

   b. number of permutations with E sixth and B last = \(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 120\)

   \(P(\text{E sixth and B last}) = \frac{\text{number of permutations with E sixth and B last}}{\text{total number of permutations}} = \frac{120}{5040} = \frac{1}{42}\)
c. \( P(C, D, B, A, G, F, E) = \frac{\text{number of permutations with order } C, D, B, A, G, F, E}{\text{total number of permutations}} = \frac{1}{5040} \)

d. \( P(\text{F or G first}) = \frac{\text{number of permutations with F or G first}}{\text{total number of permutations}} = \frac{1440}{5040} = \frac{2}{7} \)

5. a. \( 9C_3 = \frac{9!}{(9-3)!3!} = \frac{9!}{6! \cdot 3!} = \frac{9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 3 \cdot 2 \cdot 1} = 84 \)

b. \( 5C_3 = \frac{5!}{(5-3)!3!} = \frac{5!}{2! \cdot 3!} = \frac{5 \cdot 4 \cdot 3!}{2 \cdot 1 \cdot 3!} = 10 \)

c. \( P(\text{all women}) = \frac{\text{number of ways to select 3 women}}{\text{total number of possible combinations}} = \frac{10}{84} = \frac{5}{42} \)

6. a. \( 11C_4 = \frac{11!}{(11-4)!4!} = \frac{11!}{7! \cdot 4!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 3 \cdot 2 \cdot 1} = 330 \)

b. \( 6C_4 = \frac{6!}{(6-4)!4!} = \frac{6!}{2! \cdot 4!} = \frac{6 \cdot 5 \cdot 4!}{2 \cdot 1 \cdot 4!} = 15 \)

c. \( P(\text{all Republicans}) = \frac{\text{number of ways to select 4 Republicans}}{\text{total number of possible combinations}} = \frac{15}{330} = \frac{1}{22} \)

7. \( 56C_5 \times 46 = \frac{56!}{(56-5)!5!} \times 46 = \frac{56!}{51!5!} \times 46 = \frac{56 \cdot 55 \cdot 54 \cdot 53 \cdot 52 \cdot 51!}{51!5! \cdot 3 \cdot 2 \cdot 1} \times 46 = 3,819,816 \times 46 = 175,711,536 \)

\( P(\text{winning}) = \frac{\text{number of ways of winning}}{\text{total number of possible combinations}} = \frac{1}{175,711,536} \)

8. Total number of combinations:

\( 56C_5 \times 46 = \frac{56!}{(56-5)!5!} \times 46 = \frac{56!}{51!5!} \times 46 = \frac{56 \cdot 55 \cdot 54 \cdot 53 \cdot 52 \cdot 51!}{51!5! \cdot 3 \cdot 2 \cdot 1} \times 46 = 3,819,816 \times 46 = 175,711,536 \)

Number of selections that match 4 out of 5 white balls and the gold Mega Ball:

\[ \text{match 4 of the 5 selected white balls} \times \text{non-selected white balls} \times \text{gold Mega Ball} \]

\[ 5C_4 \times 51C_1 \times 1 = 5 \times 51 \times 1 = 255 \]

\( P(\text{matching 4 of the 5 white balls and the gold Mega Ball}) = \frac{255}{175,711,536} = \frac{85}{58,570,512} \)

9. Total number of combinations:

\( 56C_5 \times 46 = \frac{56!}{(56-5)!5!} \times 46 = \frac{56!}{51!5!} \times 46 = \frac{56 \cdot 55 \cdot 54 \cdot 53 \cdot 52 \cdot 51!}{51!5! \cdot 3 \cdot 2 \cdot 1} \times 46 = 3,819,816 \times 46 = 175,711,536 \)

Number of selections that match 3 out of 5 white balls and the gold Mega Ball:

\[ \text{match 3 of the 5 selected white balls} \times \text{any 2 of the 51 non-selected white balls} \times \text{gold Mega Ball} \]

\[ 5C_3 \times 51C_2 \times 1 = 10 \times 1275 \times 1 = 12,750 \]

\( P(\text{matching 3 of the 5 white balls and the gold Mega Ball}) = \frac{12,750}{175,711,536} = \frac{2125}{29,285,256} \)
10. Total number of combinations:
\[ \binom{56}{5} = \frac{56!}{51!5!} = \frac{56!}{51!5!} = \frac{56 \cdot 55 \cdot 54 \cdot 53 \cdot 52 \cdot 5!}{51! \cdot 5!} = \frac{3,819,816 \times 46}{175,711,536} = 14,875 \]

Number of selections that match 2 out of 5 white balls and the gold Mega Ball:

\[ \binom{5}{2} \times \binom{51}{3} \times 1 = 10 \times 20,825 \times 1 = 208,250 \]

\[ P( \text{matching 2 of the 5 white balls and the gold Mega Ball} ) = \frac{208,250}{175,711,536} = \frac{14,875}{12,550,824} \]

11. a. \[ \binom{25}{6} = \frac{25!}{19!6!} = \frac{25!}{19!6!} = \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19!}{19! \cdot 6!} = \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19!}{19! \cdot 6!} = 177,100 \]

\[ P(\text{all are defective}) = \frac{\text{number of ways to choose 6 defective transistors}}{\text{total number of possible combinations}} = \frac{1}{177,100} = 0.00000565 \]

b. \[ \binom{19}{6} = \frac{19!}{13!6!} = \frac{19!}{13!6!} = \frac{19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13!}{13! \cdot 6!} = \frac{19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13!}{13! \cdot 6!} = 27,132 \]

\[ P(\text{none are defective}) = \frac{\text{number of ways to choose 6 good transistors}}{\text{total number of possible permutations}} = \frac{27,132}{177,100} = \frac{969}{6325} = 0.153 \]

12. a. \[ \binom{13}{5} = \frac{13!}{8!5!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8! \cdot 5!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8! \cdot 5!} = 1287 \]

\[ \binom{6}{5} = \frac{6!}{1!5!} = \frac{6!}{1!5!} = \frac{6 \cdot 5!}{1!5!} = 6 \]

\[ P(\text{all lawyers}) = \frac{\text{number of ways to select 5 lawyers}}{\text{total number of possible combinations}} = \frac{6}{1287} = \frac{2}{429} = 0.00466 \]

b. \[ \binom{7}{2} = \frac{7!}{2!5!} = \frac{7!}{2!5!} = \frac{7 \cdot 6 \cdot 5!}{2! \cdot 5!} = 21 \]

\[ P(\text{none are lawyers}) = \frac{\text{number of ways to select 5 teachers}}{\text{total number of possible combinations}} = \frac{21}{1287} = \frac{7}{429} = 0.0163 \]

13. total number of possible combinations: \[ \binom{10}{3} = \frac{10!}{7!3!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 3!} = 120 \]

number of ways to select one Democrat: \[ \binom{6}{1} = \frac{6!}{5!} = \frac{6!}{5!} = \frac{6 \cdot 5!}{5!} = 6 \]

number of ways to select two Republicans: \[ \binom{4}{2} = \frac{4!}{2!2!} = \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2!}{2! \cdot 2!} = 6 \]

number of ways to select one Democrat and two Republicans: \[ \binom{6}{1} \cdot \binom{4}{2} = 6 \cdot 6 = 36 \]

\[ P(\text{one Democrat and two Republicans}) = \frac{36}{120} = \frac{3}{10} = 0.3 \]
14. total number of possible combinations: \( _{20}C_4 = \frac{20!}{(20-4)!4!} = \frac{20!}{16!4!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16!}{16! \cdot 3 \cdot 2 \cdot 1} = 4845 \)

number of ways to select two parents: \( _{15}C_2 = \frac{15!}{(15-2)!2!} = \frac{15!}{13!2!} = \frac{15 \cdot 14 \cdot 13!}{13!2 \cdot 1} = 105 \)

number of ways to select two teachers: \( _5C_2 = \frac{5!}{(5-2)!2!} = \frac{5!}{3!2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 10 \)

number of ways to select two parents and two teachers: \( _{15}C_2 \cdot _5C_2 = 105 \cdot 10 = 1050 \)

\[ P(\text{two parents and two teachers}) = \frac{1050}{4845} = \frac{70}{323} = 0.217 \]

15. a. \( _{52}C_5 = \frac{52!}{(52-5)!5!} = \frac{52!}{47!5!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47!}{47! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2,598,960 \)

b. \( _{13}C_5 = \frac{13!}{(13-5)!5!} = \frac{13!}{8!5!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 1287 \)

c. \[ P(\text{diamond flush}) = \frac{\text{number of possible 5-card diamond flushes}}{\text{total number of possible combinations}} = \frac{1287}{2,598,960} = 0.000495 \]

16. a. \( _{52}C_4 = \frac{52!}{(52-4)!4!} = \frac{52!}{48!4!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47!}{48! \cdot 3 \cdot 2 \cdot 1} = 2,598,960 \)

b. \( _4C_4 = \frac{4!}{(4-4)!4!} = \frac{4!}{0!4!} = 1 \)

c. \( _4C_1 = \frac{4!}{(4-1)!1!} = \frac{4!}{3!1!} = \frac{4 \cdot 3!}{3!1} = 4 \)

d. \( _4C_4 \cdot _4C_1 = 1 \cdot 4 = 4 \)

e. \[ P(\text{4 aces and 1 king}) = \frac{\text{number of hands with 4 aces and 1 king}}{\text{total number of possible combinations}} = \frac{4}{2,598,960} = 0.00000154 \]

17. total number of possible combinations: \( _{52}C_3 = \frac{52!}{(52-3)!3!} = \frac{52!}{49!3!} = \frac{52 \cdot 51 \cdot 50 \cdot 49!}{49! \cdot 2 \cdot 1} = 22,100 \)

number of ways to select 3 picture cards: \( _{12}C_3 = \frac{12!}{(12-3)!3!} = \frac{12!}{9!3!} = \frac{12 \cdot 11 \cdot 10 \cdot 9!}{9! \cdot 2 \cdot 1} = 220 \)

\[ P(\text{3 picture cards}) = \frac{220}{22,100} = \frac{11}{1105} = 0.00995 \]

18. total number of possible combinations: \( _{52}C_4 = \frac{52!}{(52-4)!4!} = \frac{52!}{48!4!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48!}{48! \cdot 3 \cdot 2 \cdot 1} = 270,725 \)

number of ways to select 4 hearts: \( _{13}C_4 = \frac{13!}{(13-4)!4!} = \frac{13!}{9!4!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9!}{9! \cdot 3 \cdot 2 \cdot 1} = 715 \)

\[ P(\text{all 4 are hearts}) = \frac{715}{270,725} = \frac{11}{4165} = 0.00264 \]
19. total number of possible combinations: \( \binom{52}{4} = \frac{52!}{(52-4)!4!} = \frac{52!}{48!4!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48!}{48! \cdot 3 \cdot 2 \cdot 1} = 270,725 \)

number of ways to select 2 queens: \( \binom{4}{2} = \frac{4!}{(4-2)!2!} = \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2!}{2! \cdot 1} = 6 \)

number of ways to select 2 kings: \( \binom{4}{2} = 6 \)

number of ways to select 2 queens and 2 kings: \( \binom{4}{2} \cdot \binom{4}{2} = 6 \cdot 6 = 36 \)

\[ P(2 \text{ queens and 2 kings}) = \frac{36}{270,725} = 0.000133 \]

20. total number of possible combinations: \( \binom{52}{4} = \frac{52!}{(52-4)!4!} = \frac{52!}{48!4!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48!}{48! \cdot 3 \cdot 2 \cdot 1} = 270,725 \)

number of ways to select 3 jacks: \( \binom{4}{3} = \frac{4!}{(4-3)!3!} = \frac{4!}{1!3!} = \frac{4 \cdot 3!}{1 \cdot 3!} = 4 \)

number of ways to select 1 queen: \( \binom{4}{1} = \frac{4!}{(4-1)!1!} = \frac{4!}{3!1!} = \frac{4 \cdot 3!}{3!1} = 4 \)

number of ways to select 3 jacks and 1 queen: \( \binom{4}{3} \cdot \binom{4}{1} = 4 \cdot 4 = 16 \)

\[ P(3 \text{ jacks and 1 queen}) = \frac{16}{270,725} = 0.000059 \]

24. makes sense

25. does not make sense; Explanations will vary. Sample explanation: Each possible combination is equally likely.

26. makes sense

27. makes sense

28. total number of possible combinations: \( 3 \cdot 2 \cdot 2 \cdot 3 = 36 \)

number of combinations the person wants: \( 1 \cdot 1 \cdot 1 \cdot 2 = 2 \)

\[ P(\text{what the person wants is available}) = \frac{\text{number of combinations the person wants}}{\text{total number of possible combinations}} = \frac{2}{36} = \frac{1}{18} \]

29. Other players could also purchase the winning combination and share the prize money, possibly making this person’s share of the prize less than what this person paid for the tickets.

30. total number of possible combinations:

\[
\begin{array}{ccc}
\text{Digit 1:} & \text{Digit 2:} & \text{Digit 3:} \\
5 & 4 & 3 \\
\times & \times & = 60
\end{array}
\]

number of even numbers greater than 500:

\[
\begin{array}{ccc}
\text{Digit 1:} & \text{Digit 2:} & \text{Digit 3:} \\
5 & 1, 3, \text{ and 2 or 4} & 2 \text{ or 4} \\
\times & \times & = 6
\end{array}
\]

\[ P(\text{even and greater than 500}) = \frac{\text{number of even numbers greater than 500}}{\text{total number of possible combinations}} = \frac{6}{60} = \frac{1}{10} \]
31. total number of possible combinations: \( \binom{52}{3} = \frac{52!}{(52-3)!3!} = \frac{52!}{49! \cdot 3!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47!}{47! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2,598,960 \)

number of ways to select one ace: \( \binom{4}{1} = \frac{4!}{(4-1)!1!} = \frac{4!}{3!} = 4 \)

Note: one card is an ace, so the other four must not be aces.

number of ways to select 4 cards with no face cards and no aces:

\( \binom{36}{4} = \frac{36!}{(36-4)!4!} = \frac{36!}{32! \cdot 4!} = \frac{36 \cdot 35 \cdot 34 \cdot 33 \cdot 32!}{32! \cdot 4! \cdot 32!} = 58,905 \)

number of hands with one ace and no face cards: \( \binom{4}{1} \cdot \binom{36}{3} = 4 \cdot 58,905 = 235,620 \)

\[ P(\text{one ace and no face cards}) = \frac{\text{number of hands with one ace and no face cards}}{\text{total number of possible combinations}} = \frac{235,620}{2,598,960} = 0.0907 \]

Check Points 11.6

1. \[ P(\text{not a diamond}) = 1 - P(\text{diamond}) = 1 - \frac{13}{52} = \frac{39}{52} = \frac{3}{4} \]

2. a. \[ P(\text{not 50 - 59}) = 1 - P(50 - 59) = 1 - \frac{31}{191} = \frac{160}{191} \]

b. \[ P(\text{at least 20 years old}) = 1 - P(\text{less than 20 years}) = 1 - \frac{9}{191} = \frac{182}{191} \]

3. \[ P(4 \text{ or 5}) = P(4) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3} \]

4. \[ P(\text{math or psychology}) = P(\text{math}) + P(\text{psychology}) - P(\text{math and psychology}) = \frac{23}{50} + \frac{11}{50} - \frac{7}{50} = \frac{27}{50} \]

5. \[ P(\text{odd or less than 5}) = P(\text{odd}) + P(\text{less than 5}) - P(\text{odd and less than 5}) = \frac{4}{8} + \frac{4}{8} - \frac{2}{8} = \frac{6}{8} = \frac{3}{4} \]

6. a. These events are not mutually exclusive.

\[ P(\text{married or female}) = P(\text{married}) + P(\text{female}) - P(\text{married and female}) \]

\[ = \frac{130}{242} + \frac{124}{242} - \frac{65}{242} = \frac{189}{242} = 0.78 \]

b. These events are mutually exclusive.

\[ P(\text{divorced or widowed}) = P(\text{divorced}) + P(\text{widowed}) \]

\[ = \frac{24}{242} + \frac{14}{242} = \frac{38}{242} = \frac{19}{121} = 0.16 \]
7. There are 2 red queens. Number of favorable outcomes = 2, Number of unfavorable outcomes = 50
   a. Odds in favor of getting a red queen are 2 to 50 or 2:50 which reduces to 1:25.
   b. Odds against getting a red queen are 50 to 2 or 50:2 which reduces to 25:1.
8. Number of unfavorable outcomes = 995, number of favorable outcomes = 5
   Odds against winning the scholarship are 995 to 5 or 995:5 which reduces to 199:1.
9. Number of unfavorable outcomes = 15, number of favorable outcomes = 1
   Odds in favor of the horse winning the race are 1 to 15
   \[ P(\text{the horse wins race}) = \frac{1}{1+15} = \frac{1}{16} = 0.0625 \text{ or } 6.3\% . \]

Concept and Vocabulary Check 11.6
1. \[ 1 - P(E) ; 1 - P(\text{not } E) \]
2. mutually exclusive; \( P(A) + P(B) \)
3. \( P(A) + P(B) - P(A \text{ and } B) \)
4. \( E \text{ will occur}; E \text{ will not occur} \)
5. \( P(E) = \frac{a}{a+b} \)
6. false
7. false
8. true
9. false

Exercise Set 11.6
1. \( P(\text{not an ace}) = 1 - P(\text{ace}) = 1 - \frac{4}{52} = \frac{48}{52} = \frac{12}{13} \)
2. \( P(\text{not a 3}) = 1 - P(3) = 1 - \frac{4}{52} = 1 - \frac{1}{13} = \frac{12}{13} \)
3. \( P(\text{not a heart}) = 1 - P(\text{heart}) = 1 - \frac{13}{52} = \frac{39}{52} = \frac{3}{4} \)
4. \( P(\text{not a club}) = 1 - P(\text{club}) = 1 - \frac{13}{52} = \frac{39}{52} = \frac{3}{4} \)
5. \( P(\text{not a picture card}) = 1 - P(\text{picture card}) = 1 - \frac{12}{52} = \frac{40}{52} = \frac{10}{13} \)
6. \( P( \text{not a red picture card} ) = 1 - P( \text{red picture card} ) = 1 - \frac{6}{52} = \frac{46}{52} = \frac{23}{26} \)

7. \( P( \text{not a straight flush} ) = 1 - P( \text{straight flush} ) = 1 - \frac{36}{2,598,960} = \frac{2,598,924}{2,598,960} = 0.999986 \)

8. \( P( \text{not four of a kind} ) = 1 - P( \text{four of a kind} ) = 1 - \frac{624}{2,598,960} = \frac{2,598,336}{2,598,960} = 0.999760 \)

9. \( P( \text{not a full house} ) = 1 - P( \text{full house} ) = 1 - \frac{3744}{2,598,960} = \frac{2,595,216}{2,598,960} = 0.998559 \)

10. \( P( \text{not a flush} ) = 1 - P( \text{flush} ) = 1 - \frac{5108}{2,598,960} = \frac{2,593,852}{2,598,960} = 0.998035 \)

11. a. 0.10 (read from graph)
    b. \( 1.00 - 0.10 = 0.90 \)

12. a. 0.78 (read from graph)
    b. \( 1.00 - 0.78 = 0.22 \)

13. \( P( \text{not age 25 - 44} ) = 1 - P( \text{age 25 - 44} ) = 1 - \frac{1080}{3000} = \frac{1920}{3000} = \frac{16}{25} \)

14. \( P( \text{not age 45 - 64} ) = 1 - P( \text{age 45 - 64} ) = 1 - \frac{840}{3000} = \frac{2160}{3000} = \frac{18}{25} \)

15. \( P( \text{age less than 65} ) = 1 - P( \text{age 65 - 74} ) = 1 - \frac{180}{3000} = \frac{2820}{3000} = \frac{47}{50} \)

16. \( P( \text{age at least 25} ) = 1 - P( \text{age 12 - 24} ) = 1 - \frac{900}{3000} = \frac{2100}{3000} = \frac{7}{10} \)

17. \( P(2 \text{ or 3} ) = P(2) + P(3) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13} \)

18. \( P(7 \text{ or 8} ) = P(7) + P(8) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13} \)

19. \( P(\text{red 2 or black 3} ) = P(\text{red 2}) + P(\text{black 3} ) = \frac{2}{52} + \frac{2}{52} = \frac{4}{52} = \frac{1}{13} \)

20. \( P(\text{red 7 or black 8} ) = P(\text{red 7}) + P(\text{black 8} ) = \frac{2}{52} + \frac{2}{52} = \frac{4}{52} = \frac{1}{13} \)

21. \( P(2 \text{ of hearts or 3 of spades} ) = P(2 \text{ of hearts}) + P(3 \text{ of spades} ) = \frac{1}{52} + \frac{1}{52} = \frac{2}{52} = \frac{1}{26} \)

22. \( P(7 \text{ or hearts or 8 of spades} ) = P(7 \text{ of hearts}) + P(8 \text{ of spades} ) = \frac{1}{52} + \frac{1}{52} = \frac{2}{52} = \frac{1}{26} \)

Copyright © 2015 Pearson Education, Inc. 493
23. \( P(\text{professor or instructor}) = P(\text{professor}) + P(\text{instructor}) = \frac{8}{44} + \frac{10}{44} = \frac{18}{44} = \frac{9}{22} \)

24. \( P(\text{Independent or Green}) = P(\text{Independent}) + P(\text{Green}) = \frac{8}{67} + \frac{4}{67} = \frac{12}{67} \)

25. \( P(\text{even or less than 5}) = P(\text{even}) + P(\text{less than 5}) - P(\text{even and less than 5}) = \frac{3}{6} + \frac{4}{6} - \frac{2}{6} = \frac{5}{6} \)

26. \( P(\text{odd or less than 4}) = P(\text{odd}) + P(\text{less than 4}) - P(\text{odd and less than 4}) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6} = \frac{2}{3} \)

27. \( P(\text{7 or red}) = P(\text{7}) + P(\text{red}) - P(\text{red 7}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13} \)

28. \( P(\text{5 or black}) = P(\text{5}) + P(\text{black}) - P(\text{black 5}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13} \)

29. \( P(\text{heart or picture card}) = P(\text{heart}) + P(\text{picture card}) - P(\text{heart and picture card}) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26} \)

30. \( P(\text{greater than 2 and less than 7, or diamond}) \\
= P(\text{greater than 2 and less than 7}) + P(\text{diamond}) - P(\text{diamond greater than 2 and less than 7}) \\
= \frac{16}{52} + \frac{13}{52} - \frac{4}{52} = \frac{25}{52} \)

31. \( P(\text{odd or less than 6}) = P(\text{odd}) + P(\text{less than 6}) - P(\text{odd and less than 6}) = \frac{4}{8} + \frac{5}{8} - \frac{3}{8} = \frac{6}{8} = \frac{3}{4} \)

32. \( P(\text{odd or greater than 3}) = P(\text{odd}) + P(\text{greater than 3}) - P(\text{odd and greater than 3}) = \frac{4}{8} + \frac{5}{8} - \frac{2}{8} = \frac{7}{8} \)

33. \( P(\text{even or greater than 5}) = P(\text{even}) + P(\text{greater than 5}) - P(\text{even and greater than 5}) = \frac{4}{8} + \frac{3}{8} - \frac{2}{8} = \frac{5}{8} \)

34. \( P(\text{even or less than 4}) = P(\text{even}) + P(\text{less than 4}) - P(\text{even and less than 4}) = \frac{4}{8} + \frac{3}{8} - \frac{1}{8} = \frac{6}{8} = \frac{3}{4} \)

35. \( P(\text{professor or male}) = P(\text{professor}) + P(\text{male}) - P(\text{male professor}) = \frac{19}{40} + \frac{22}{40} - \frac{8}{40} = \frac{33}{40} \)

36. \( P(\text{professor or female}) = P(\text{professor}) + P(\text{female}) - P(\text{female professor}) = \frac{19}{40} + \frac{18}{40} - \frac{11}{40} = \frac{26}{40} = \frac{13}{20} \)

37. \( P(\text{teach. assist. or female}) = P(\text{teach. assist.}) + P(\text{female}) - P(\text{female teach. assist.}) = \frac{21}{40} + \frac{18}{40} - \frac{7}{40} = \frac{32}{40} = \frac{4}{5} \)

38. \( P(\text{teaching assistant or male}) = P(\text{teach. assist.}) + P(\text{male}) - P(\text{male teach. assist.}) = \frac{21}{40} + \frac{22}{40} - \frac{14}{40} = \frac{29}{40} \)
39. \[ P(\text{Democrat or business major}) = P(\text{Democrat}) + P(\text{business major}) - P(\text{Democrat and business major}) \]
\[ = \frac{29}{50} + \frac{11}{50} - \frac{5}{50} = \frac{35}{50} = \frac{7}{10} \]

40. \[ P(\text{math or english}) = P(\text{math}) + P(\text{english}) - P(\text{math and english}) = \frac{135}{200} + \frac{85}{200} - \frac{65}{200} = \frac{155}{200} = \frac{31}{40} \]

41. \[ P(\text{not completed 4 years or more}) = 1 - P(\text{completed 4 years or more}) = 1 - \frac{45}{174} = \frac{129}{174} = \frac{43}{58} \]

42. \[ P(\text{not completed 4 years of high school}) = \frac{29}{174} = \frac{1}{6} \]

43. \[ P(\text{completed 4 years of high school only or less than four years of college}) = P(\text{completed 4 years of high school only}) + P(\text{less than four years of college}) \]
\[ = \frac{56}{174} + \frac{44}{174} = \frac{100}{174} = \frac{50}{87} \]

44. \[ P(\text{completed less than 4 years of high school or 4 years of high school only}) = P(\text{completed less than 4 years of high school}) + P(\text{4 years of high school only}) \]
\[ = \frac{29}{174} + \frac{56}{174} = \frac{85}{174} \]

45. \[ P(\text{completed 4 years of high school only or is a man}) = P(\text{completed 4 years of high school only}) + P(\text{male}) - P(\text{completed 4 years of high school only and is a man}) \]
\[ = \frac{56}{174} + \frac{82}{174} - \frac{25}{174} = \frac{113}{174} \]

46. \[ P(\text{completed 4 years of high school only or is a woman}) = P(\text{completed 4 years of high school only}) + P(\text{female}) - P(\text{completed 4 years of high school only and is a woman}) \]
\[ = \frac{56}{174} + \frac{92}{174} - \frac{31}{174} = \frac{117}{174} = \frac{39}{58} \]

47. The number that meets the characteristic is 45. The number that does not meet the characteristic is 174 - 45 = 129. Odds in favor: 45 to 129 which reduces to 15 to 43
Odds against: 129 to 45 which reduces to 43 to 5

48. The number that meets the characteristic is 29. The number that does not meet the characteristic is 174 - 29 = 145. Odds in favor: 29 to 145 which reduces to 1 to 5
Odds against: 145 to 29 which reduces to 5 to 1

49. \[ P(\text{not in the Army}) = 1 - P(\text{in the Army}) = 1 - \frac{490 + 80}{1420} = 1 - \frac{570}{1420} = \frac{85}{142} \]

50. \[ P(\text{not in the Marines}) = 1 - P(\text{in the Marines}) = 1 - \frac{190 + 10}{1420} = 1 - \frac{200}{1420} = \frac{61}{71} \]
51. \[ P(\text{in the Navy or a man}) = P(\text{in the Navy}) + P(\text{a man}) - P(\text{in the Navy and a man}) \]
\[ = \frac{270 + 50}{1420} + \frac{270 + 490 + 190 + 270}{1420} - \frac{270}{1420} \]
\[ = \frac{320 + 1220}{1420} - \frac{270}{1420} \]
\[ = \frac{1270}{1420} - \frac{270}{1420} \]
\[ = \frac{1270}{1420} - \frac{270}{1420} \]
\[ = \frac{127}{142} \]

52. \[ P(\text{in the Army or a woman}) = P(\text{in the Army}) + P(\text{a woman}) - P(\text{in the Army and a woman}) \]
\[ = \frac{490 + 80}{1420} + \frac{60 + 80 + 10 + 50}{1420} - \frac{80}{1420} \]
\[ = \frac{570 + 200}{1420} - \frac{80}{1420} \]
\[ = \frac{690}{1420} - \frac{80}{1420} \]
\[ = \frac{690}{1420} - \frac{80}{1420} \]
\[ = \frac{69}{142} \]

53. \[ P(\text{in the Air Force or the Marines}) = P(\text{in the Air Force}) + P(\text{in the Marines}) = \frac{270 + 60}{1420} + \frac{190 + 10}{1420} = \frac{530}{1420} = \frac{53}{142} \]

54. \[ P(\text{in the Army or the Navy}) = P(\text{in the Army}) + P(\text{in the Navy}) = \frac{490 + 80}{1420} + \frac{270 + 50}{1420} - \frac{890}{1420} - \frac{89}{142} \]

55. The number that meets the characteristic is 270 + 50 = 320.
The number that does not meet the characteristic is 1420 – 320 = 1100.
Odds in favor: 320 to 1100 which reduce 16 to 55
Odds against: 1100 to 320 which reduce 55 to 16

56. The number that meets the characteristic is 490 + 80 = 570.
The number that does not meet the characteristic is 1420 – 570 = 850.
Odds in favor: 570 to 850 which reduce 57 to 85
Odds against: 850 to 570 which reduce 85 to 57

57. The number that meets the characteristic is 10. The number that does not meet the characteristic is 1420 – 10 = 1410.
Odds in favor: 10 to 1410 which reduce 1 to 141
Odds against: 1410 to 10 which reduce 141 to 1

58. The number that meets the characteristic is 60. The number that does not meet the characteristic is 1420 – 60 = 1360.
Odds in favor: 60 to 1360 which reduce 1 to 68
Odds against: 1360 to 60 which reduce 68 to 1

59. The number that meets the characteristic is 270 + 490 + 190 + 270 = 1220.
The number that does not meet the characteristic is 1420 – 1220 = 200.
Odds in favor: 1220 to 200 which reduce 61 to 10
Odds against: 200 to 1220 which reduce 10 to 61

60. The number that meets the characteristic is 60 + 80 + 10 + 50 = 200.
The number that does not meet the characteristic is 1420 – 200 = 1220.
Odds favor: 200 to 1220 which reduce 10 to 61
Odds in against: 1220 to 200 which reduce 61 to 10
61. number of favorable outcomes = 4, number of unfavorable outcomes = 2
   Odds in favor of getting a number greater than 2 are 4:2, or 2:1.

62. number of favorable outcomes = 4, number of unfavorable outcomes = 2
   Odds in favor of getting a number less than 5 are 4:2, or 2:1.

63. number of unfavorable outcomes = 2, number of favorable outcomes = 4
   Odds against getting a number greater than 2 or 2:4, or 1:2.

64. number of unfavorable outcomes = 2, number of favorable outcomes = 4
   Odds against getting a number less than 5 are 2:4, or 1:2.

65. number of favorable outcomes = 9, number of unfavorable outcomes = 100 – 9 = 91
   a. Odds in favor of a child in a one-parent household having a parent who is a college graduate are 9:91.
   b. Odds against a child in a one-parent household having a parent who is a college graduate are 91:9.

66. number of favorable outcomes = 29, number of unfavorable outcomes = 100 – 29 = 71
   a. Odds in favor of a child in a two-parent household having parents who are college graduates are 29:71.
   b. Odds against a child in a two-parent household having parents who are college graduates are 71:29.

67. number of favorable outcomes = 13, number of unfavorable outcomes = 39
   Odds in favor of a heart are 13:39, or 1:3.

68. number of favorable outcomes = 12, number of unfavorable outcomes = 40
   Odds in favor of a picture card are 12:40, or 3:10.

69. number of favorable outcomes = 26, number of unfavorable outcomes = 26
   Odds in favor of a red card are 26:26, or 1:1.

70. number of favorable outcomes = 26, number of unfavorable outcomes = 26
   Odds in favor of a black card are 26:26, or 1:1.

71. number of unfavorable outcomes = 48, number of favorable outcomes = 4
   Odds against a 9 are 48:4, or 12:1.

72. number of unfavorable outcomes = 48, number of favorable outcomes = 4
   Odds against a 5 are 48:4, or 12:1.

73. number of unfavorable outcomes = 50, number of favorable outcomes = 2
   Odds against a black king are 50:2, or 25:1.

74. number of unfavorable outcomes = 50, number of favorable outcomes = 2
   Odds against a red jack are 50:2, or 25:1.

75. number of unfavorable outcomes = 47, number of favorable outcomes = 5
   Odds against a spade greater than 3 and less than 9 are 47:5.

76. number of unfavorable outcomes = 47, number of favorable outcomes = 5
   Odds against a club greater than 4 and less than 9 are 47:5.

77. number of unfavorable outcomes = 980, number of favorable outcomes = 20
   Odds against winning are 980:20, or 49:1.

78. number of unfavorable outcomes = 4970, number of favorable outcomes = 30
   Odds against winning are 4970:30, or 497:3.
79. The number that meets the characteristic is 18. The number that does not meet the characteristic is $38 - 18 = 20$. Odds in favor: 18 to 20 which reduce 9 to 10

80. The number that meets the characteristic is 10. The number that does not meet the characteristic is $38 - 10 = 28$. Odds in favor: 10 to 28 which reduce 5 to 14

81. The number that meets the characteristic is 10. The number that does not meet the characteristic is $38 - 10 = 28$. Odds against: 28 to 10 which reduce 14 to 5

82. The number that meets the characteristic is 18. The number that does not meet the characteristic is $38 - 18 = 20$. Odds against: 20 to 18 which reduce 10 to 9

83. The number that meets the characteristic is $18 + 10 = 28$. The number that does not meet the characteristic is $38 - 28 = 10$. Odds in favor: 28 to 10 which reduce 14 to 5

84. The number that meets the characteristic is $10 + 10 = 20$. The number that does not meet the characteristic is $38 - 20 = 18$. Odds in favor: 20 to 18 which reduce 10 to 9

85. The number that meets the characteristic is $10 + 10 = 20$. The number that does not meet the characteristic is $38 - 20 = 18$. Odds against: 18 to 20 which reduce 9 to 10

86. The number that meets the characteristic is $18 + 10 = 28$. The number that does not meet the characteristic is $38 - 28 = 10$. Odds against: 10 to 28 which reduce 5 to 14

87. \[ P(\text{winning}) = \frac{3}{3 + 4} = \frac{3}{7} \]

88. \[ P(\text{winning}) = \frac{3}{3 + 7} = \frac{3}{10} \]

89. \[ P(\text{miss free throw}) = \frac{4}{21 + 4} = \frac{4}{25} = 0.16 = 16\% \]
   In 100 free throws, on average he missed 16, so he made $100 - 16 = 84$.

90. \[ P(\text{still alive at age 70}) = \frac{193}{193 + 270} = \frac{193}{463} = 41.7\% \]

91. \[ P(\text{contracting an airborne illness}) = \frac{1}{1 + 999} = \frac{1}{1000} \]

92. \[ P(\text{deep-vein thrombosis}) = \frac{1}{1 + 28} = \frac{1}{29} \]

100. does not make sense; Explanations will vary. Sample explanation: The two probabilities must add to 1.

101. does not make sense; Explanations will vary. Sample explanation: Since 1 card is a heart and a king, the probability is $\frac{4}{52} + \frac{13}{52} = \frac{16}{52} = \frac{4}{13}$.

102. does not make sense; Explanations will vary. Sample explanation: The probability of selecting a king or a heart is $\frac{4}{52} + \frac{13}{52} = \frac{16}{52} = \frac{4}{13}$. The probability of selecting the king of hearts is $\frac{1}{52}$. 

Copyright © 2015 Pearson Education, Inc.
103. does not make sense; Explanations will vary. Sample explanation: The odds are more likely 1:9.

104. a. \( P(\text{Democrat who is not a business major}) \)
   \[ = 1 - P(\text{not a Democrat or Democrat and business major}) \]
   \[ = 1 - [P(\text{not a Democrat}) + P(\text{Democrat and business major})] \]
   \[ = 1 - \left( \frac{21}{50} + \frac{5}{50} \right) \]
   \[ = 1 - \frac{26}{50} \]
   \[ = \frac{24}{50} \]
   \[ = \frac{12}{25} \]

b. \( P(\text{neither Democrat nor business major}) \)
   \[ = 1 - P(\text{Democrat or business major}) \]
   \[ = 1 - [P(\text{Democrat}) + P(\text{business major}) - P(\text{Democrat and business major})] \]
   \[ = 1 - \left( \frac{29}{50} + \frac{11}{50} - \frac{5}{50} \right) \]
   \[ = 1 - \frac{35}{50} \]
   \[ = \frac{15}{50} \]
   \[ = \frac{3}{10} \]

105. \( P(\text{driving intoxicated or driving accident}) \)
   \[ = P(\text{driving intoxicated}) + P(\text{driving accident}) - P(\text{driving accident while intoxicated}) \]

   Substitute the three given probabilities and solve for the unknown probability:
   \[ 0.35 = 0.32 + 0.09 - P(\text{driving accident while intoxicated}) \]
   \( P(\text{driving accident while intoxicated}) = 0.32 + 0.09 - 0.35 \)
   \( P(\text{driving accident while intoxicated}) = 0.06 \)

106. \[ \frac{P(E)}{1 - P(E)} = \frac{a}{b} \]
   \[ \frac{P(E)}{1 - P(E)} \cdot \frac{b(1 - P(E))}{1} = \frac{a}{b} \cdot \frac{b(1 - P(E))}{1} \]
   \[ bP(E) = a(1 - P(E)) \]
   \[ bP(E) = a - aP(E) \]
   \[ aP(E) + bP(E) = a \]
   \[ P(E)(a + b) = a \]
   \[ \frac{P(E)(a + b)}{a + b} = \frac{a}{a + b} \]
   \[ P(E) = \frac{a}{a + b} \]
Check Points 11.7

1. \( P(\text{green and green}) = P(\text{green}) \cdot P(\text{green}) = \frac{2}{38} \cdot \frac{2}{38} = \frac{1}{19} \cdot \frac{1}{19} = \frac{1}{361} = 0.0027738 \)

2. \( P(4 \text{ boys in a row}) = P(\text{boy and boy and boy and boy}) = P(\text{boy}) \cdot P(\text{boy}) \cdot P(\text{boy}) \cdot P(\text{boy}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16} \)

3. a. \( P(\text{hit four years in a row}) = P(\text{hit}) \cdot P(\text{hit}) \cdot P(\text{hit}) \cdot P(\text{hit}) = \frac{5}{19} \cdot \frac{5}{19} \cdot \frac{5}{19} = \frac{125}{130,321} = 0.005 \)

   b. Note: \( P(\text{not hit in any single year}) = 1 - P(\text{hit in any single year}) = 1 - \frac{5}{19} = \frac{14}{19} \). Therefore, \( P(\text{not hit in next four years}) = \frac{14}{19} \cdot \frac{14}{19} \cdot \frac{14}{19} \cdot \frac{14}{19} = \frac{43,046,731}{130,321} = 0.295 \)

   c. \( P(\text{hit at least once in next four years}) = 1 - P(\text{not hit in next four years}) = 1 - \frac{43,046,731}{130,321} = \frac{87,274,590}{130,321} = 0.705 \)

4. \( P(2 \text{ kings}) = P(\text{king}) \cdot P(\text{king given the first card was a king}) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{17} \cdot \frac{1}{221} = 0.00452 \)

5. \( P(3 \text{ hearts}) = P(\text{heart}) \cdot P(\text{heart given the first card was a heart}) \cdot P(\text{heart given the first two cards were hearts}) = \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} = \frac{1}{17} \cdot \frac{11}{50} = 0.0129 \)

6. The sample space is given by \( S = \{a, e, i, o, u\} \).
   Of these 5 elements, only a and e precede h.
   Thus the probability is \( P(\text{letter precedes h | vowel}) = \frac{2}{5} \).

7. a. The sample space is the set of 13 spades.
   Of these 13 elements, all 13 cards are black.
   Thus the probability is \( P(\text{black card | spade}) = \frac{13}{13} = 1 \).

   b. The sample space is the set of 26 black cards.
   Of these 26 elements, 13 cards are spades.
   Thus the probability is \( P(\text{spade | black card}) = \frac{13}{26} = \frac{1}{2} \).

8. a. \( P(\text{positive mammogram | breast cancer}) = \frac{720}{800} = \frac{9}{10} = 0.9 \).

   b. \( P(\text{breast cancer | positive mammogram}) = \frac{720}{7664} = \frac{45}{479} = 0.094 \).
Concept and Vocabulary Check 11.7

1. independent; \( P(A) \cdot P(B) \)

2. the event does not occur

3. dependent; \( P(A) \cdot P(B \text{ given that } A \text{ occurred}) \)

4. conditional; \( P(B|A) \)

5. false

6. false

7. true

8. true

Exercise Set 11.7

1. \( P(\text{green and then red}) = P(\text{green}) \cdot P(\text{red}) = \frac{2}{6} \cdot \frac{3}{6} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{6} \)

2. \( P(\text{yellow and then green}) = P(\text{yellow}) \cdot P(\text{green}) = \frac{1}{6} \cdot \frac{2}{6} = \frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18} \)

3. \( P(\text{yellow and then yellow}) = P(\text{yellow}) \cdot P(\text{yellow}) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \)

4. \( P(\text{red and then red}) = P(\text{red}) \cdot P(\text{red}) = \frac{3}{6} \cdot \frac{3}{6} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \)

5. \( P(\text{color other than red each time}) = P(\text{not red}) \cdot P(\text{not red}) = \frac{3}{6} \cdot \frac{3}{6} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \)

6. \( P(\text{color other than green each time}) = P(\text{not green}) \cdot P(\text{not green}) = \frac{4}{6} \cdot \frac{4}{6} = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9} \)

7. \( P(\text{green and then red and then yellow}) = P(\text{green}) \cdot P(\text{red}) \cdot P(\text{yellow}) = \frac{2}{6} \cdot \frac{3}{6} \cdot \frac{1}{6} = \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{36} \)

8. \( P(\text{red and then red and then green}) = P(\text{red}) \cdot P(\text{red}) \cdot P(\text{green}) = \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{2}{6} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{12} \)

9. \( P(\text{red every time}) = P(\text{red}) \cdot P(\text{red}) \cdot P(\text{red}) = \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6} = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} \)

10. \( P(\text{green every time}) = P(\text{green}) \cdot P(\text{green}) \cdot P(\text{green}) = \frac{2}{6} \cdot \frac{2}{6} \cdot \frac{2}{6} = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{27} \)
11. \( P(2 \text{ and then } 3) = P(2) \cdot P(3) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \)

12. \( P(5 \text{ and then } 1) = P(5) \cdot P(1) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \)

13. \( P(\text{even and then greater than } 2) = P(\text{even}) \cdot P(\text{greater than } 2) = \frac{3}{6} \cdot \frac{4}{6} = \frac{2}{3} = \frac{1}{3} \)

14. \( P(\text{odd and then less than } 3) = P(\text{odd}) \cdot P(\text{less than } 3) = \frac{3}{6} \cdot \frac{2}{3} = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3} \)

15. \( P(\text{picture card and then heart}) = P(\text{picture card}) \cdot P(\text{heart}) = \frac{12}{52} \cdot \frac{13}{52} = \frac{3}{13} \cdot \frac{1}{4} = \frac{3}{52} \)

16. \( P(\text{jack and then club}) = P(\text{jack}) \cdot P(\text{club}) = \frac{4}{52} \cdot \frac{13}{52} = \frac{1}{13} \cdot \frac{1}{4} = \frac{1}{52} \)

17. \( P(2 \text{ kings}) = P(\text{king}) \cdot P(\text{king}) = \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{13} \cdot \frac{1}{13} = \frac{1}{169} \)

18. \( P(3 \text{ each time}) = P(3) \cdot P(3) = \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{13} \cdot \frac{1}{13} = \frac{1}{169} \)

19. \( P(\text{red each time}) = P(\text{red}) \cdot P(\text{red}) = \frac{26}{52} \cdot \frac{26}{52} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \)

20. \( P(\text{black each time}) = P(\text{black}) \cdot P(\text{black}) = \frac{26}{52} \cdot \frac{26}{52} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \)

21. \( P(\text{all heads}) = P(\text{heads}) \cdot P(\text{heads}) \cdot P(\text{heads}) \cdot P(\text{heads}) \cdot P(\text{heads}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{64} \)

22. \( P(\text{all tails}) = P(\text{tails}) \cdot P(\text{tails}) \cdot P(\text{tails}) \cdot P(\text{tails}) \cdot P(\text{tails}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{128} \)

23. \( P(\text{head and number greater than } 4) = P(\text{head}) \cdot P(\text{number greater than } 4) = \frac{1}{2} \cdot \frac{2}{6} = \frac{1}{2} \cdot \frac{2}{6} = \frac{1}{6} \)

24. \( P(\text{tail and number less than } 5) = P(\text{tail}) \cdot P(\text{number less than } 5) = \frac{1}{2} \cdot \frac{4}{6} = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3} \)

25. a. \( P(\text{hit two years in a row}) = P(\text{hit}) \cdot P(\text{hit}) = \frac{1}{16} \cdot \frac{1}{16} = \frac{1}{256} = 0.00391 \)

   b. \( P(\text{Hit three consecutive years}) = P(\text{hit}) \cdot P(\text{hit}) \cdot P(\text{hit}) = \frac{1}{16} \cdot \frac{1}{16} \cdot \frac{1}{16} = \frac{1}{4096} = 0.000244 \)

   c. \( P(\text{not hit in next ten years}) = [P(\text{not hit})]^{10} = \left(1 - \frac{1}{16}\right)^{10} = \left(\frac{15}{16}\right)^{10} = 0.524 \)

   d. \( P(\text{hit at least once in next ten years}) = 1 - P(\text{not hit in next ten years}) = 1 - 0.524 = 0.476 \)
26. a. \( P(\text{flood two years in a row}) = P(\text{flood}) \cdot P(\text{flood}) = \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100} \)

b. \( P(\text{flood three consecutive years}) = P(\text{flood}) \cdot P(\text{flood}) \cdot P(\text{flood}) = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{1000} \)

c. \( P(\text{no flooding in ten years}) = [P(\text{no flood})]^{10} = [1 - P(\text{flood})]^{10} = \left(1 - \frac{1}{10}\right)^{10} = \left(\frac{9}{10}\right)^{10} = 0.349 \)

d. \( P(\text{flooding at least once in ten years}) = 1 - P(\text{no flooding in ten years}) = 1 - 0.349 = 0.651 \)

27. \( P(\text{both suffer from depression - from general population}) = P(\text{depression}) \cdot P(\text{depression}) = 0.12 \cdot 0.12 = 0.0144 \)

28. \( P(\text{both suffer from depression - from population of smokers}) = P(\text{depression}) \cdot P(\text{depression}) = 0.28 \cdot 0.28 = 0.0784 \)

29. \( P(\text{all three suffer from frequent hangovers - from population of smokers}) = P(\text{frequent hangovers}) \cdot P(\text{frequent hangovers}) \cdot P(\text{frequent hangovers}) = 0.20 \cdot 0.20 \cdot 0.20 = 0.008 \)

30. \( P(\text{all three suffer from frequent hangovers - from general population}) = P(\text{frequent hangovers}) \cdot P(\text{frequent hangovers}) \cdot P(\text{frequent hangovers}) = 0.10 \cdot 0.10 \cdot 0.10 = 0.001 \)

31. \( P(\text{at least one of three suffers from anxiety/panic disorder - from population of smokers}) \)
\[ = 1 - \left[1 - P(\text{anxiety/panic disorder})\right] \cdot \left[1 - P(\text{anxiety/panic disorder})\right] \cdot \left[1 - P(\text{anxiety/panic disorder})\right] \]
\[ = 1 - [1 - 0.19] \cdot [1 - 0.19] \cdot [1 - 0.19] \]
\[ = 1 - [0.81] \cdot [0.81] \cdot [0.81] \]
\[ = 1 - 0.5314 \]
\[ = 0.4686 \]

32. \( P(\text{at least one of three suffers from severe pain - from population of smokers}) \)
\[ = 1 - \left[1 - P(\text{severe pain})\right] \cdot \left[1 - P(\text{severe pain})\right] \cdot \left[1 - P(\text{severe pain})\right] \]
\[ = 1 - [1 - 0.14] \cdot [1 - 0.14] \cdot [1 - 0.14] \]
\[ = 1 - [0.86] \cdot [0.86] \cdot [0.86] \]
\[ = 1 - 0.6361 \]
\[ = 0.3639 \]

33. \( P(\text{solid and solid}) = P(\text{solid}) \cdot P(\text{solid given first was solid}) = \frac{15}{30} \cdot \frac{14}{29} = \frac{1}{2} \cdot \frac{14}{29} = \frac{7}{29} \)

34. \( P(\text{two caramel}) = P(\text{caramel}) \cdot P(\text{caramel given first was caramel}) = \frac{10}{30} \cdot \frac{9}{29} = \frac{9}{3} \cdot \frac{9}{29} = \frac{3}{29} \)

35. \( P(\text{coconut then caramel}) = P(\text{coconut}) \cdot P(\text{caramel given first was coconut}) = \frac{5}{30} \cdot \frac{10}{29} = \frac{1}{6} \cdot \frac{10}{29} = \frac{5}{87} \)

36. \( P(\text{coconut then solid}) = P(\text{coconut}) \cdot P(\text{solid given first was coconut}) = \frac{5}{30} \cdot \frac{15}{29} = \frac{1}{6} \cdot \frac{15}{29} = \frac{5}{58} \)

37. \( P(\text{two Democrats}) = P(\text{Democrat}) \cdot P(\text{Democrat given first was Democrat}) = \frac{5}{15} \cdot \frac{4}{14} = \frac{1}{3} \cdot \frac{2}{7} = \frac{2}{21} \)
38.  \( P(\text{two Republicans}) = P(\text{Republican}) \cdot P(\text{Republican given first was Republican}) \)
\[
= \frac{6}{15} \cdot \frac{5}{14} = \frac{2}{5} \cdot \frac{5}{14} = \frac{1}{7}
\]

39.  \( P(\text{Independent then Republican}) = P(\text{Independent}) \cdot P(\text{Republican given first was Independent}) \)
\[
= \frac{4}{15} \cdot \frac{6}{14} = \frac{4}{15} \cdot \frac{3}{7} = \frac{4}{35}
\]

40.  \( P(\text{Independent then Democrat}) = P(\text{Independent}) \cdot P(\text{Democrat given first was Independent}) \)
\[
= \frac{4}{15} \cdot \frac{5}{14} = 2 \cdot \frac{2}{21}
\]

41.  \( P(\text{no Independents}) = P(\text{not Independent}) \cdot P(\text{not Independent given first was not Independent}) \)
\[
= \frac{11}{15} \cdot \frac{10}{14} = \frac{11}{15} \cdot \frac{5}{7} = \frac{11}{21}
\]

42.  \( P(\text{no Democrats}) = P(\text{not Democrat}) \cdot P(\text{not Democrat given first was not Democrat}) \)
\[
= \frac{10}{15} \cdot \frac{9}{14} = \frac{2}{3} \cdot \frac{9}{14} = \frac{3}{7}
\]

43.  \( P(\text{three cans of apple juice}) \)
\[
= P(\text{apple juice}) \cdot P(\text{apple juice given first was apple juice}) \cdot P(\text{apple juice given first two were apple juice}) \]
\[
= \frac{6}{20} \cdot \frac{5}{19} \cdot \frac{4}{18} = \frac{1}{57}
\]

44.  \( P(\text{three cans of grape juice}) \)
\[
= P(\text{grape juice}) \cdot P(\text{grape juice given first was grape juice}) \cdot P(\text{grape juice given first two were grape juice}) \]
\[
= \frac{8}{20} \cdot \frac{7}{19} \cdot \frac{6}{18} = \frac{14}{285}
\]

45.  \( P(\text{grape juice then orange juice then mango juice}) \)
\[
= P(\text{grape juice}) \cdot P(\text{orange juice given first was grape juice}) \cdot P(\text{mango juice given first was grape juice and second was orange juice}) \]
\[
= \frac{8}{20} \cdot \frac{4}{19} \cdot \frac{2}{18} = \frac{8}{855}
\]

46.  \( P(\text{apple juice then grape juice then orange juice}) \)
\[
= P(\text{apple juice}) \cdot P(\text{grape juice given first was apple juice}) \cdot P(\text{orange juice given first was apple juice and second was grape juice}) \]
\[
= \frac{6}{20} \cdot \frac{8}{19} \cdot \frac{4}{18} = \frac{8}{285}
\]

47.  \( P(\text{no grape juice}) \)
\[
= P(\text{not grape juice}) \cdot P(\text{not grape juice given first was not grape juice}) \cdot P(\text{not grape juice given first two were not grape juice}) \]
\[
= \frac{12}{20} \cdot \frac{11}{19} \cdot \frac{10}{18} = \frac{11}{57}
\]

48.  \( P(\text{no apple juice}) \)
\[
= P(\text{not apple juice}) \cdot P(\text{not apple juice given first was not apple juice}) \cdot P(\text{not apple juice given first two were not apple juice}) \]
\[
= \frac{14}{20} \cdot \frac{13}{19} \cdot \frac{12}{18} = \frac{91}{285}
\]

49.  \( P(\text{red}) = \frac{1}{5} \)

50.  \( P(\text{yellow}) = \frac{1}{3} \)
51. \( P(\text{even} | \text{yellow}) = \frac{2}{3} \)

52. \( P(\text{odd} | \text{red}) = \frac{3}{5} \)

53. \( P(\text{red} | \text{odd}) = \frac{3}{4} \)

54. \( P(\text{yellow} | \text{odd}) = \frac{1}{4} \)

55. \( P(\text{red} | \text{at least } 5) = \frac{3}{4} \)

56. \( P(\text{yellow} | \text{at most } 3) = \frac{1}{3} \)

57. \( P(\text{surviving} | \text{wore seat belt}) = \frac{412,368}{412,878} = \frac{68,728}{68,813} = 0.999 \)

58. \( P(\text{not surviving} | \text{did not wear seat belt}) = \frac{1601}{164,128} = 0.010 \)

59. \( P(\text{wore seat belt} | \text{driver survived}) = \frac{412,368}{574,895} = 0.717 \)

60. \( P(\text{did not wear seat belt} | \text{not surviving}) = \frac{1601}{2111} = 0.758 \)

61. \( P(\text{not divorced}) = 1 - P(\text{divorced}) = 1 - \frac{24}{242} = \frac{218}{242} = \frac{109}{121} = 0.90 \)

62. \( P(\text{not widowed}) = 1 - P(\text{widowed}) = 1 - \frac{14}{242} = \frac{228}{242} = \frac{114}{121} = 0.94 \)

63. \( P(\text{widowed or divorced}) = P(\text{widowed}) + P(\text{divorced}) = \frac{14}{242} + \frac{24}{242} = \frac{38}{242} = \frac{19}{121} = 0.16 \)

64. \( P(\text{never married or is divorced}) = P(\text{never married}) + P(\text{divorced}) = \frac{74}{242} + \frac{24}{242} = \frac{98}{242} = \frac{49}{121} = 0.40 \)

65. \( P(\text{male or is divorced}) = P(\text{male}) + P(\text{divorced}) - P(\text{male and is divorced}) = \frac{118}{242} + \frac{24}{242} - \frac{10}{242} = \frac{132}{242} = \frac{6}{11} = 0.55 \)

66. \( P(\text{female or is divorced}) = P(\text{female}) + P(\text{divorced}) - P(\text{female and is divorced}) = \frac{124}{242} + \frac{24}{242} - \frac{14}{242} = \frac{134}{242} = \frac{67}{121} = 0.55 \)

67. \( P(\text{male} | \text{divorced}) = \frac{10}{24} = \frac{5}{12} = 0.42 \)
68. \( P(\text{female | divorced}) = \frac{14}{24} = \frac{7}{12} = 0.58 \)

69. \( P(\text{widowed | woman}) = \frac{11}{124} = 0.09 \)

70. \( P(\text{divorced | man}) = \frac{10}{118} = \frac{5}{59} = 0.08 \)

71. \( P(\text{never married or married | man}) = \frac{40}{118} + \frac{65}{118} = \frac{105}{118} = 0.89 \)

72. \( P(\text{never married or married | woman}) = \frac{34}{124} + \frac{65}{124} = \frac{99}{124} = 0.80 \)

73. a. The first person can have any of 365 birthdays. To not match, the second person can then have any of the remaining 364 birthdays.

b. \( P(\text{three different birthdays}) = \frac{365 \cdot 364 \cdot 363}{365 \cdot 365 \cdot 365} = 0.992 \)

c. \( P(\text{at least two have same birthday}) = 1 - P(\text{three different birthdays}) = 1 - 0.992 = 0.008 \)

d. \( P(\text{20 different birthdays}) = \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} \cdot \frac{360}{365} \cdot \frac{359}{365} \cdot \frac{358}{365} \cdot \frac{357}{365} \cdot \frac{356}{365} \cdot \frac{355}{365} \cdot \frac{354}{365} \cdot \frac{353}{365} \cdot \frac{352}{365} \cdot \frac{351}{365} \cdot \frac{350}{365} \cdot \frac{349}{365} \cdot \frac{348}{365} \cdot \frac{347}{365} \cdot \frac{346}{365} = 0.589 \)

\( P(\text{at least two have same birthday}) = 1 - P(\text{20 different birthdays}) = 1 - 0.589 = 0.411 \)

e. 23 people (determine by trial-and-error using method shown in part d)

\( P(\text{23 different birthdays}) = \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} \cdot \frac{360}{365} \cdot \frac{359}{365} \cdot \frac{358}{365} \cdot \frac{357}{365} \cdot \frac{356}{365} \cdot \frac{355}{365} \cdot \frac{354}{365} \cdot \frac{353}{365} \cdot \frac{352}{365} \cdot \frac{351}{365} \cdot \frac{350}{365} \cdot \frac{349}{365} \cdot \frac{348}{365} \cdot \frac{347}{365} \cdot \frac{346}{365} \cdot \frac{345}{365} \cdot \frac{344}{365} \cdot \frac{343}{365} = 0.493 \)

\( P(\text{at least two have same birthday}) = 1 - P(\text{23 different birthdays}) = 1 - 0.493 = 0.507 \)

82. does not make sense; Explanations will vary. Sample explanation: The previous three children do not affect the odds of the fourth child. The odds are 1:1.

83. does not make sense; Explanations will vary. Sample explanation: The probability of the second selection being a man, given that the first was a man is \( \frac{4}{9} \).

84. makes sense

85. does not make sense; Explanations will vary. Sample explanation: \( P(A | B) \) does not necessarily equal \( P(B | A) \).

86. \( P(\text{no one hospitalized}) = [P(\text{not hospitalized})]^5 = (0.9)(0.9)(0.9)(0.9)(0.9) = (0.9)^5 = 0.59049 = 59.0\% \)

87. \( P(\text{2 on 1st, 3rd, and 4th rolls only}) = P(2) \cdot P(\text{not 2}) \cdot P(2) \cdot P(\text{not 2}) = \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{25}{7776} = 0.00322 \)
88. There are 5 odd numbered cards and therefore there are \( \binom{5}{2} = 10 \) ways to get two odd cards.
There are 4 even numbered cards and therefore there are \( \binom{4}{2} = 6 \) ways to get two even cards.
Note that the sum of two odds is an even number and that the sum of two evens is also an even number.
Since selecting one even card and one odd card would result in an odd sum, we only need to consider the 16 possible outcomes calculated above.
\[
P\left( \text{both odd|sum even} \right) = \frac{\text{number of outcomes with both odd and even sum}}{\text{number of outcomes where the sum is even}} = \frac{10}{16} = \frac{5}{8}
\]

89. The sample space has 36 elements. Of these elements, the following 11 fit the given condition: 1&5, 1&6, 3&5, 3&6, 5&1, 5&3, 5&5, 5&6, 6&1, 6&3, 6&5. Thus the probability is \( \frac{11}{36} \).

Check Points 11.8
1. \( E = \frac{1}{4} \cdot 1 + \frac{1}{4} + 3 \cdot \frac{1}{4} + 4 \cdot \frac{1}{4} = \frac{1+2+3+4}{4} = \frac{10}{4} = 2.5 \)
2. \( E = 0 \cdot \frac{1}{16} + 1 \cdot \frac{4}{16} + 2 \cdot \frac{6}{16} + 3 \cdot \frac{4}{16} + 4 \cdot \frac{1}{16} = \frac{0+4+12+12+4}{16} = \frac{32}{16} = 2 \)
3. a. \( E = -0.01 \cdot 1 + 0.2 \cdot 2 + 0.08 \cdot 3 + 0.05 \cdot 4 + 0.01 \cdot 5 + 0.7 \cdot 10 = 8 \) This means that in the long run, the average cost of a claim is expected to be $8000.
   b. An average premium charge of $8000 would cause the company to neither lose nor gain money.
4. \( E = \left( \frac{1}{5} \right) + \left( -\frac{1}{4} \right) \cdot \frac{4}{5} = \frac{1}{5} + \left( -\frac{1}{5} \right) = 0 \)
   Since the expected value is 0, there is nothing to gain or lose on average by guessing.
5. Values of gain or loss:
   Grand Prize: $1000 – $2 = $998 ,
   Consolation Prize: $50 – $2 = $48 ,
   Nothing: $0 – $2 = $–2
   \( E = (-2) \left( \frac{997}{1000} \right) + (48) \left( \frac{2}{1000} \right) + (998) \left( \frac{1}{1000} \right) = \frac{-1994 + 96 + 998}{1000} = -\frac{900}{1000} = -0.90 \)
   The expected value for one ticket is $–0.90 . This means that in the long run a player can expect to lose $0.90 for each ticket bought. Buying five tickets will make your likelihood of winning five times greater, however there is no advantage to this strategy because the cost of five tickets is also five times greater than one ticket.
6. \( E = (2.20) \left( \frac{20}{80} \right) + (-1.00) \left( \frac{60}{80} \right) = \frac{44 - 60}{80} = \frac{-16}{80} = -0.20 \)
   This means that in the long run a player can expect to lose an average of $0.20 for each $1 bet.

Concept and Vocabulary Check 11.8
1. expected; probability; add
2. loss; probability; add
3. false
4. true
Exercise Set 11.8

1. \[ E = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{4} = 1.75 \]

2. \[ E = 1 \cdot \frac{1}{8} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{2} + 4 \cdot \frac{1}{4} = 2.875 \]

3. a. \[ E = 0(0.65) + 50,000(0.20) + 100,000(0.10) + 150,000(0.03) + 200,000(0.01) + 250,000(0.01) = 29,000 \]
   This means that in the long run the average cost of a claim is $29,000.

   b. $29,000

   c. $29,050

4. a. \[ E = 0(0.70) + 20,000(0.20) + 40,000(0.06) + 60,000(0.02) + 80,000(0.01) + 100,000(0.01) = 9400 \]
   This means that in the long run the average cost of a claim is $9400.

   b. $9400

   c. $9450

5. \[ E = -10,000(0.9) + 90,000(0.1) = 0 \]
   This means on the average there will be no gain or loss.

6. \[ E = -1500 \left( \frac{4}{5} \right) + 38,500 \left( \frac{1}{5} \right) = 6500 \]
   This means an expected gain on the average.

7. \[ E = -99,999 \left( \frac{27}{10,000,000} \right) + 1 \left( \frac{9,999,973}{10,000,000} \right) = 0.73 \]

8. \[ E = -9,900(0.002) + 100(0.998) = 80 \]

9. Probabilities after eliminating one possible answer: Guess Correctly: \( \frac{1}{4} \), Guess Incorrectly: \( \frac{3}{4} \)
   \[ E = (1) \left( \frac{1}{4} \right) + \left( -\frac{1}{4} \right) \left( \frac{3}{4} \right) = \frac{1}{4} + \left( -\frac{3}{16} \right) = \frac{1}{16} \]
   expected points on a guess if one answer is eliminated.
   Yes, it is advantageous to guess after eliminating one possible answer.

10. Probabilities after eliminating two possible answers: Guess Correctly: \( \frac{1}{3} \), Guess Incorrectly: \( \frac{2}{3} \)
    \[ E = (1) \left( \frac{1}{3} \right) + \left( -\frac{1}{4} \right) \left( \frac{2}{3} \right) = \frac{1}{3} + \left( -\frac{1}{6} \right) = \frac{1}{6} \]
    expected points on a guess if two answers are eliminated.
    Yes, it is advantageous to guess after eliminating two possible answers.

11. First mall: \[ E = 300,000 \left( \frac{1}{2} \right) - 100,000 \left( \frac{1}{2} \right) = 100,000 \]
    Second mall: \[ E = 200,000 \left( \frac{3}{4} \right) - 60,000 \left( \frac{1}{4} \right) = 135,000 \]
    Choose the second mall.
12. Site A: 
\[ E = 80(0.2) - 10(0.8) = 8 \text{ million} \]  
Site B: 
\[ E = 120(0.1) - 18(0.9) = -4.2 \text{ million} \]  
Site A has the larger expected profit. 
$8 \text{ million} - (-4.2 \text{ million}) = 12.2 \text{ million} \]  
Site A’s profit exceeds Site B’s by 12.2 million.

13. a. 
\[ E = 700,000(0.2) + 0(0.8) = 140,000 \]  

b. No

14. 
\[ E = 80\left(\frac{99}{100}\right) - 270\left(\frac{1}{100}\right) = 76.50 \]  

15. 
\[ E = 4\left(\frac{1}{6}\right) - 1\left(\frac{5}{6}\right) = -\frac{1}{6} = -0.17 \]  
This means an expected loss of approximately $0.17 per game.

16. 
\[ E = -0.25\left(\frac{1}{6}\right) + 0.75\left(\frac{1}{6}\right) + 1.75\left(\frac{1}{6}\right) - 1.25\left(\frac{3}{6}\right) = -0.25 \]  
This means an expected loss of $0.25 per game.

17. 
\[ E = 1\left(\frac{18}{38}\right) - 1\left(\frac{20}{38}\right) = -0.053 \]  
This means an expected loss of approximately $0.053 per $1.00 bet.

18. 
\[ E = 4\left(\frac{3}{10}\right) + 2\left(\frac{1}{10}\right) - 2\left(\frac{4}{10}\right) - 3\left(\frac{2}{10}\right) = 0 \]  
A player should expect to break even.

19. 
\[ E = 499\left(\frac{1}{1000}\right) - 1\left(\frac{999}{1000}\right) = -0.50 \]  
This means an expected loss of $0.50 per $1.00 bet.

26. does not make sense; Explanations will vary. Sample explanation: The expectation does not need to be a whole number.

27. makes sense

28. does not make sense; Explanations will vary. Sample explanation: The likely outcome of playing longer is more losses.

29. does not make sense; Explanations will vary. Sample explanation: The expected value of a lottery game is less than the cost of the ticket.

30. First determine the probabilities.

\[ \text{Total number of possible combinations} = \binom{35}{5} = \frac{35!}{30!5!} = 324,632 \]  
\[ \text{Number of ways to select all 5} = \binom{5}{5} = 1 \]  
\[ \text{Number of ways to select 4 of the 5 winning numbers and 1 of the 30 losing numbers} = \binom{5}{4} \times \binom{30}{1} = 5 \times 30 = 150 \]  
\[ \text{Number of ways to select 3 of the 5 winning numbers and 2 of the 30 losing numbers} = \binom{5}{3} \times \binom{30}{2} = 10 \times 435 = 4350 \]  
\[ P(\text{all 5}) = \frac{1}{324,632}; \quad P(4 \text{ of 5}) = \frac{150}{324,632}; \quad P(3 \text{ of 5}) = \frac{4350}{324,632}; \quad P(\text{losing}) = \frac{324,632 - 150 - 4350}{324,632} = \frac{320,131}{324,632} \]  
\[ E = 49,999\left(\frac{1}{324,632}\right) + 499\left(\frac{150}{324,632}\right) + 4\left(\frac{4350}{324,632}\right) - 1\left(\frac{320,131}{324,632}\right) = -0.55 \]  
This means an expected loss of $0.55 per $1.00 ticket.
31. Let \( x \) = the charge for the policy. Note, the expected value, \( E = $60 \).
\[
E = (x - $200,000)(0.0005) + (x)(0.9995) = $60 = 0.0005x - $100 + 0.9995x
\]
\[
$160 = x
\]
The insurance company should charge $160 for the policy.

Chapter 11 Review Exercises

1. Use the Fundamental Counting Principle with two groups of items. \( 20 \cdot 40 = 800 \)
2. Use the Fundamental Counting Principle with two groups of items. \( 4 \cdot 5 = 20 \)
3. Use the Fundamental Counting Principle with two groups of items. \( 100 \cdot 99 = 9900 \)
4. Use the Fundamental Counting Principle with three groups of items. \( 5 \cdot 5 \cdot 5 = 125 \)
5. Use the Fundamental Counting Principle with five groups of items. \( 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243 \)
6. Use the Fundamental Counting Principle with four groups of items. \( 5 \cdot 2 \cdot 2 \cdot 3 = 60 \)
7. \[
\frac{16!}{14!} = \frac{16 \cdot 15 \cdot 14!}{14!} = 240
\]
8. \[
\frac{800!}{799!} = \frac{800 \cdot 799}{799!} = 800
\]
9. \( 5!-3! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 - 3 \cdot 2 \cdot 1 = 120 - 6 = 114 \)
10. \[
\frac{11!}{(11-3)!} = \frac{11!}{8!} = \frac{11 \cdot 10 \cdot 9 \cdot 8!}{8!} = \frac{990}{8} = 114
\]
11. \[
\frac{10!}{(10-6)!} = \frac{10!}{4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!} = 151,200
\]
12. \[
\frac{100!}{(100-2)!} = \frac{100!}{98!} = \frac{100 \cdot 99 \cdot 98!}{98!} = 9900
\]
13. \[
\frac{11!}{(11-7)!} = \frac{11!}{4!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 7!} = 330
\]
14. \[
\frac{14!}{(14-5)!} = \frac{14!}{9!} = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9!}{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 2002
\]
15. Order does not matter. This problem involves combinations.
16. Order matters. This problem involves permutations.
17. Order does not matter. This problem involves combinations.
18. Use the Fundamental Counting Principle with six groups of items. \( 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720 \)
19. \[
\frac{15!}{(15-4)!} = \frac{15!}{11!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11!}{11!} = 32,760
\]
20. \( \binom{10}{4} = \frac{10!}{(10-4)!4!} = \frac{10!}{6!4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 210 \)

21. \( \frac{n!}{p!q!} = \frac{7!}{3!2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 2!} = 420 \)

22. \( \binom{20}{3} = \frac{20!}{(20-3)!3!} = \frac{20!}{17!3!} = \frac{20 \cdot 19 \cdot 18 \cdot 17!}{17! \cdot 3 \cdot 2 \cdot 1} = 1140 \)

23. Use the Fundamental Counting Principle with seven groups of items. \( 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 120 \)

24. \( P_5 = \frac{20!}{(20-5)!} = \frac{20!}{15!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15!}{15!} = 1,860,480 \)

25. Use the Fundamental Counting Principle with five groups of items. \( 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \)

26. \( \binom{13}{5} = \frac{13!}{(13-5)!5!} = \frac{13!}{8!5!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 1,287 \)

27. Choose the Republicans: \( \binom{12}{5} = \frac{12!}{(12-5)!5!} = \frac{12!}{7!5!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{7! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 792 \)

Choose the Democrats: \( \binom{8}{4} = \frac{8!}{(8-4)!4!} = \frac{8!}{4!4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3 \cdot 2 \cdot 1} = 70 \)

Multiply the choices: \( 792 \cdot 70 = 55,440 \)

28. \( \frac{n!}{p!q!} = \frac{6!}{3!2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 2!} = 60 \)

29. \( P(6) = \frac{\text{number of ways a 6 can occur}}{\text{total number of possible outcomes}} = \frac{1}{6} \)

30. \( P(\text{less than 5}) = \frac{\text{number of ways a number less than 5 can occur}}{\text{total number of possible outcomes}} = \frac{4}{6} = \frac{2}{3} \)

31. \( P(\text{less than 7}) = \frac{\text{number of ways a number less than 7 can occur}}{\text{total number of possible outcomes}} = \frac{6}{6} = 1 \)

32. \( P(\text{greater than 6}) = \frac{\text{number of ways a number greater than 6 can occur}}{\text{total number of possible outcomes}} = \frac{0}{6} = 0 \)

33. \( P(5) = \frac{\text{number of ways a 5 can occur}}{\text{total number of possible outcomes}} = \frac{4}{52} = \frac{1}{13} \)

34. \( P(\text{picture card}) = \frac{\text{number of ways a picture card can occur}}{\text{total number of possible outcomes}} = \frac{12}{52} = \frac{3}{13} \)

35. \( P(\text{greater than 4 and less than 8}) = \frac{\text{number of ways a card greater than 4 and less than 8 can occur}}{\text{total number of possible outcomes}} = \frac{12}{52} = \frac{3}{13} \)
36. \( P(\text{4 of diamonds}) = \frac{\text{number of ways a 4 of diamonds can occur}}{\text{total number of possible outcomes}} = \frac{1}{52} \)

37. \( P(\text{red ace}) = \frac{\text{number of ways a red ace can occur}}{\text{total number of possible outcomes}} = \frac{2}{52} = \frac{1}{26} \)

38. \( P(\text{chocolate}) = \frac{\text{number of ways a chocolate can occur}}{\text{total number of possible outcomes}} = \frac{15}{30} = \frac{1}{2} \)

39. \( P(\text{caramel}) = \frac{\text{number of ways a caramel can occur}}{\text{total number of possible outcomes}} = \frac{10}{30} = \frac{1}{3} \)

40. \( P(\text{peppermint}) = \frac{\text{number of ways a peppermint can occur}}{\text{total number of possible outcomes}} = \frac{5}{30} = \frac{1}{6} \)

41. a. \( P(\text{carrier without the disease}) = \frac{\text{number of ways to be a carrier without the disease}}{\text{total number of possible outcomes}} = \frac{2}{4} = \frac{1}{2} \)
   
b. \( P(\text{disease}) = \frac{\text{number of ways to have the disease}}{\text{total number of possible outcomes}} = \frac{0}{4} = 0 \)

42. \( P(\text{employed}) = \frac{140}{240} = \frac{7}{12} \)

43. \( P(\text{female}) = \frac{124}{240} = \frac{31}{60} \)

44. \( P(\text{unemployed male}) = \frac{8}{240} = \frac{1}{30} \)

45. number of ways to visit in order D, B, A, C = 1
   total number of possible permutations = \(4 \cdot 3 \cdot 2 \cdot 1 = 24\)
   \( P(D, B, A, C) = \frac{1}{24} \)

46. number of permutations with C last = \(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120\)
   total number of possible permutations = \(6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720\)
   \( P(C \text{ last}) = \frac{120}{720} = \frac{1}{6} \)

47. number of permutations with B first and A last = \(1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 24\)
   total number of possible permutations = \(6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720\)
   \( P(\text{B first and A last}) = \frac{24}{720} = \frac{1}{30} \)

48. number of permutations in order F, E, A, D, C, B = 1
   total number of possible permutations = \(6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720\)
   \( P(F, E, A, D, C, B) = \frac{1}{720} \)
49. number of permutations with A or C first = 2 · 5 · 4 · 3 · 2 · 1 = 240
   total number of possible permutations = 6 · 5 · 4 · 3 · 2 · 1 = 720
   \[ P(A \text{ or } C \text{ first}) = \frac{240}{720} = \frac{1}{3} \]

50. a. number of ways to win = 1
   total number of possible combinations:
   \[ _{20}C_5 = \frac{20!}{(20-5)!5!} = \frac{20!}{15!5!} = \frac{20\cdot19\cdot18\cdot17\cdot16\cdot15!}{15!4\cdot3\cdot2\cdot1} = 15,504 \]
   \[ P(\text{winning with one ticket}) = \frac{1}{15,504} = 0.0000645 \]

   b. number of ways to win = 100
   \[ P(\text{winning with 100 different tickets}) = \frac{100}{15,504} = 0.00645 \]

51. a. number of ways to select 4 Democrats:
   \[ _6C_4 = \frac{6!}{(6-4)!4!} = \frac{6!}{2!4!} = \frac{6\cdot5\cdot4!}{4\cdot2!} = 15 \]
   total number of possible combinations:
   \[ _{10}C_4 = \frac{10!}{(10-4)!4!} = \frac{10!}{6!4!} = \frac{10\cdot9\cdot8\cdot7\cdot6!}{6\cdot5\cdot4\cdot3\cdot2\cdot1} = 210 \]
   \[ P(\text{all Democrats}) = \frac{15}{210} = \frac{1}{14} \]

   b. number of ways to select 2 Democrats:
   \[ _6C_2 = \frac{6!}{(6-2)!2!} = \frac{6!}{4!2!} = \frac{6\cdot5\cdot4!}{4\cdot2\cdot1} = 15 \]
   number of ways to select 2 Republicans:
   \[ _4C_2 = \frac{4!}{(4-2)!2!} = \frac{4!}{2!2!} = \frac{4\cdot3\cdot2!}{2\cdot1\cdot1} = 6 \]
   number of ways to select 2 Democrats and 2 Republicans = 15 · 6 = 90
   \[ P(2 \text{ Democrats and 2 Republicans}) = \frac{90}{210} = \frac{3}{7} \]

52. number of ways to get 2 picture cards:
   \[ _6C_2 = \frac{6!}{(6-2)!2!} = \frac{6!}{4!2!} = \frac{6\cdot5\cdot4!}{4\cdot2\cdot1} = 15 \]
   number of ways to get one non-picture card = 20
   number of ways to get 2 picture cards and one non-picture card = 15 · 20 = 300
   total number of possible combinations:
   \[ _{26}C_3 = \frac{26!}{(26-3)!3!} = \frac{26!}{23!3!} = \frac{26\cdot25\cdot24\cdot23!}{23\cdot3\cdot2\cdot1} = 2600 \]
   \[ P(2 \text{ picture cards}) = \frac{300}{2600} = \frac{3}{26} \]

53. \[ P(\text{not a 5}) = 1 - P(5) = 1 - \frac{1}{6} = \frac{5}{6} \]

54. \[ P(\text{not less than 4}) = 1 - P(\text{less than 4}) = 1 - \frac{3}{6} = 1 - \frac{1}{2} = \frac{1}{2} \]

55. \[ P(3 \text{ or } 5) = P(3) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3} \]
56. \( P(\text{less than 3 or greater than 4}) = P(\text{less than 3}) + P(\text{greater than 4}) = \frac{2}{6} + \frac{2}{6} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3} \)

57. \( P(\text{less than 5 or greater than 2}) = P(\text{less than 5}) + P(\text{greater than 2}) - P(\text{less than 5 and greater than 2}) \\
= \frac{4}{6} + \frac{4}{6} - \frac{2}{6} = 1 \)

58. \( P(\text{not a picture card}) = 1 - P(\text{picture card}) = 1 - \frac{12}{52} = 1 - \frac{3}{13} = \frac{10}{13} \)

59. \( P(\text{not a diamond}) = 1 - P(\text{diamond}) = 1 - \frac{13}{52} = 1 - \frac{1}{4} = \frac{3}{4} \)

60. \( P(\text{ace or king}) = P(\text{ace}) + P(\text{king}) = \frac{4}{52} + \frac{4}{52} = \frac{1}{13} + \frac{1}{13} = \frac{2}{13} \)

61. \( P(\text{black 6 or red 7}) = P(\text{black 6}) + P(\text{red 7}) = \frac{2}{52} + \frac{2}{52} = \frac{1}{26} + \frac{1}{26} = \frac{2}{26} = \frac{1}{13} \)

62. \( P(\text{queen or red card}) = P(\text{queen}) + P(\text{red card}) - P(\text{red queen}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13} \)

63. \( P(\text{club or picture card}) = P(\text{club}) + P(\text{picture card}) - P(\text{club and picture card}) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26} \)

64. \( P(\text{not 4}) = 1 - P(4) = 1 - \frac{1}{6} = \frac{5}{6} \)

65. \( P(\text{not yellow}) = 1 - P(\text{yellow}) = 1 - \frac{1}{6} = \frac{5}{6} \)

66. \( P(\text{not red}) = 1 - P(\text{red}) = 1 - \frac{3}{6} = 1 - \frac{1}{2} = \frac{1}{2} \)

67. \( P(\text{red or yellow}) = P(\text{red}) + P(\text{yellow}) = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3} \)

68. \( P(\text{red or even}) = P(\text{red}) + P(\text{even}) - P(\text{red and even}) = \frac{3}{6} + \frac{3}{6} - \frac{0}{6} = 1 \)

69. \( P(\text{red or greater than 3}) = P(\text{red}) + P(\text{greater than 3}) - P(\text{red and greater than 3}) = \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6} \)

70. \( P(\text{African American or male}) = P(\text{African American}) + P(\text{male}) - P(\text{African American male}) \\
= \frac{50 + 20}{200} + \frac{50 + 90}{200} - \frac{50}{200} = \frac{160}{200} = \frac{4}{5} \)

71. \( P(\text{female or white}) = P(\text{female}) + P(\text{white}) - P(\text{white female}) = \frac{20 + 40}{200} + \frac{90 + 40}{200} - \frac{40}{200} = \frac{150}{200} = \frac{3}{4} \)

72. \( P(\text{public college}) = \frac{252}{350} = \frac{18}{25} \)
73. \( P(\text{not from high-income family}) = 1 - P(\text{from high-income family}) = 1 - \frac{50}{350} = \frac{350 - 50}{350} = \frac{300}{350} = \frac{6}{7} \)

74. \( P(\text{from middle-income family or high-income family}) = \frac{160 + 50}{350} = \frac{210}{350} = \frac{3}{5} \)

75. \( P(\text{attended private college or is from a high income family}) = P(\text{private college}) + P(\text{high income family}) - P(\text{attended private college and is from a high income family}) \\
= \frac{98}{350} + \frac{50}{350} - \frac{28}{350} = \frac{120}{350} = \frac{12}{35} \)

76. number of favorable outcomes = 4, number of unfavorable outcomes = 48
   Odds in favor of getting a queen are 4:48, or 1:12. Odds against getting a queen are 12:1.

77. number of favorable outcomes = 20, number of unfavorable outcomes = 1980
   Odds against winning are 1980:20, or 99:1.

78. \( P(\text{win}) = \frac{3}{3+1} = \frac{3}{4} \)

79. \( P(\text{yellow then red}) = P(\text{yellow}) \cdot P(\text{red}) = \frac{2}{6} \cdot \frac{4}{6} = \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9} \)

80. \( P(1 \text{ then } 3) = P(1) \cdot P(3) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \)

81. \( P(\text{yellow both times}) = P(\text{yellow}) \cdot P(\text{yellow}) = \frac{2}{6} \cdot \frac{2}{6} = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9} \)

82. \( P(\text{yellow then 4 then odd}) = P(\text{yellow}) \cdot P(4) \cdot P(\text{odd}) = \frac{2}{6} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{36} \)

83. \( P(\text{red every time}) = P(\text{red}) \cdot P(\text{red}) \cdot P(\text{red}) = \frac{4}{6} \cdot \frac{4}{6} \cdot \frac{2}{3} = \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27} \)

84. \( P(\text{five boys in a row}) = P(\text{boy}) \cdot P(\text{boy}) \cdot P(\text{boy}) \cdot P(\text{boy}) \cdot P(\text{boy}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{32} \)

85. a. \( P(\text{flood two years in a row}) = P(\text{flood}) \cdot P(\text{flood}) = (0.2)(0.2) = 0.04 \)
   
   b. \( P(\text{flood for three consecutive years}) = P(\text{flood}) \cdot P(\text{flood}) \cdot P(\text{flood}) = (0.2)(0.2)(0.2) = 0.008 \)
   
   c. \( P(\text{no flooding for four consecutive years}) = [1 - P(\text{flood})]^4 = (1 - 0.2)^4 = (0.8)^4 = 0.4096 \)
   
   d. \( P(\text{flood at least once in next four years}) = 1 - P(\text{no flooding for four consecutive years}) \\
= 1 - 0.4096 = 0.5904 \)

86. \( P(\text{music major then psychology major}) = P(\text{music major}) \cdot P(\text{psychology major given first was music major}) = \frac{2}{9} \cdot \frac{4}{8} = \frac{2}{9} \cdot \frac{1}{2} = \frac{1}{9} \)

87. \( P(\text{two business majors}) = P(\text{bus. major}) \cdot P(\text{bus. major given first was bus. major}) = \frac{3}{9} \cdot \frac{2}{8} = \frac{3}{9} \cdot \frac{1}{4} = \frac{1}{12} \)
Chapter 11  Counting Methods and Probability Theory

88. \( P(\text{solid then two cherry}) \)
    \[ = P(\text{solid}) \cdot P(\text{cherry given first was solid}) \cdot P(\text{cherry given first was solid and second was cherry}) \]
    \[ = \frac{30}{50} \cdot \frac{5}{49} \cdot \frac{4}{48} = \frac{3 \cdot 5 \cdot 1}{5 \cdot 49 \cdot 12} = \frac{1}{196} \]

89. \( P(5|\text{odd}) = \frac{1}{3} \)

90. \( P(\text{vowel|precedes the letter k}) = \frac{3}{10} \)

91. a. \( P(\text{odd|red}) = \frac{2}{4} = \frac{1}{2} \)
    b. \( P(\text{yellow|at least 3}) = \frac{2}{7} \)

92. \( P(\text{does not have TB}) = \frac{11 + 124}{9 + 1 + 11 + 124} = \frac{135}{145} = \frac{27}{29} \)

93. \( P(\text{tests positive}) = \frac{9 + 11}{9 + 1 + 11 + 124} = \frac{20}{145} = \frac{4}{29} \)

94. \( P(\text{does not have TB or tests positive}) \)
    \[ = P(\text{does not have TB}) + P(\text{tests positive}) - P(\text{does not have TB and tests positive}) \]
    \[ = \frac{11 + 124}{145} + \frac{9 + 11}{145} - \frac{11}{145} \]
    \[ = \frac{144}{145} \]

95. \( P(\text{does not have TB|positive test}) = \frac{11}{9 + 11} = \frac{11}{20} \)

96. \( P(\text{tests positive|does not have TB}) = \frac{11}{11 + 124} = \frac{11}{135} \)

97. \( P(\text{has TB|negative Test}) = \frac{1}{1 + 124} = \frac{1}{125} \)

98. \( P(\text{two people with TB}) = P(\text{TB}) \cdot P(\text{TB|first person selected has TB}) \)
    \[ = \frac{10}{145} \cdot \frac{9}{144} = \frac{1}{232} \]

99. \( P(\text{two people with positive tests}) = P(\text{positive test}) \cdot P(\text{positive test|first person has positive test}) \)
    \[ = \frac{20}{145} \cdot \frac{19}{144} = \frac{19}{1044} \]

100. \( P(\text{male}) = \frac{27,336}{31,593} = 0.865 \)

101. \( P(\text{age 25-44}) = \frac{11,161}{31,593} = 0.353 \)
102.  
\[ P(\text{less than 75}) = 1 - P(\text{greater than or equal to 75}) = 1 - \frac{2379}{31,593} = \frac{29,214}{31,593} = 0.925 \]

103.  
\[ P(\text{age 20 – 24 or 25 – 44}) = P(\text{age 20 – 24}) + P(\text{age 25 – 44}) = \frac{4095}{31,593} + \frac{11,161}{31,593} = \frac{15,256}{31,593} = 0.483 \]

104.  
\[ P(\text{female or younger than 5}) = P(\text{female}) + P(\text{younger than 5}) - P(\text{female and younger than 5}) = \frac{4257}{31,593} + \frac{88}{31,593} - \frac{33}{31,593} = \frac{4312}{31,593} = 0.136 \]

105.  
\[ P(\text{age 20 – 24} | \text{male}) = \frac{3684}{27,336} = 0.135 \]

106.  
\[ P(\text{male} | \text{at least 75}) = \frac{2169}{2379} = 0.912 \]

107.  
\[ E = 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{8} + 3 \cdot \frac{1}{8} + 4 \cdot \frac{1}{4} + 5 \cdot \frac{1}{4} = 3.125 \]

108.  

a.  
\[ E = (0)(0.0000005) + (-1)(0.9999995) = -0.50 \]

   The insurance company spends an average of $0.50 per person insured.

b.  
\[ \text{charge } 9.50 - (-0.50) = 10.00 \]

109.  
\[ E = 27,000 \left(\frac{1}{4}\right) + (-3000) \left(\frac{3}{4}\right) = 4500. \text{ The expected gain is$4500 per bid.} \]

110.  
\[ E = 1 \cdot \left(\frac{2}{4}\right) + 1 \cdot \left(\frac{1}{4}\right) + (-4) \cdot \left(\frac{1}{4}\right) = -0.25. \text{ The expected loss is$0.25 per game.} \]

---

Chapter 11 Test

1.  
Use the Fundamental Counting Principle with five groups of items.  \(10 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 240\)

2.  
Use the Fundamental Counting Principle with four groups of items.  \(4 \cdot 3 \cdot 2 \cdot 1 = 24\)

3.  
Use the Fundamental Counting Principle with seven groups of items.  \(1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720\)

4.  
\[ _{11}P_3 = \frac{11!}{(11-3)!} = \frac{11!}{8!} = \frac{11 \cdot 10 \cdot 9 \cdot 8!}{8!} = 990 \]

5.  
\[ _{10}C_4 = \frac{10!}{(10-4)!4!} = \frac{10!}{6!4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 210 \]

6.  
\[ \frac{n!}{p!q!} = \frac{7!}{3!2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 2 \cdot 1} = 420 \]

7.  
\[ P(\text{freshman}) = \frac{12}{50} = \frac{6}{25} \]
8. \( P(\text{not a sophomore}) = 1 - P(\text{sophomore}) = 1 - \frac{16}{50} = 1 - \frac{8}{25} = \frac{17}{25} \)

9. \( P(\text{junior or senior}) = P(\text{junior}) + P(\text{senior}) = \frac{20}{50} + \frac{2}{50} = \frac{22}{50} = \frac{11}{25} \)

10. \( P(\text{greater than 4 and less than 10}) = \frac{20}{52} = \frac{5}{13} \)

11. \( P(\text{C first, A next-to-last, E last}) = P(\text{C}) \cdot P(\text{A given C was first}) \cdot P(\text{E given C was first and A was next-to-last}) = \frac{1}{7} \cdot \frac{1}{6} \cdot \frac{1}{5} = \frac{1}{210} \)

12. total number of possible combinations: \( \binom{15}{6} = \frac{15!}{(15-6)!6!} = \frac{15!}{9!6!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9!}{9! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 5005 \)

\( P(\text{winning with 50 tickets}) = \frac{50}{5005} = \frac{10}{1001} = 0.00999 \)

13. \( P(\text{red or blue}) = P(\text{red}) + P(\text{blue}) = \frac{2}{8} + \frac{2}{8} = \frac{4}{8} = \frac{1}{2} \)

14. \( P(\text{red then blue}) = P(\text{red}) \cdot P(\text{blue}) = \frac{2}{8} \cdot \frac{2}{8} = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16} \)

15. \( P(\text{flooding for three consecutive years}) = P(\text{flood}) \cdot P(\text{flood}) \cdot P(\text{flood}) = \frac{1}{20} \cdot \frac{1}{20} = \frac{1}{8000} \)

16. \( P(\text{black or picture card}) = P(\text{black}) + P(\text{picture card}) - P(\text{black picture card}) = \frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52} = \frac{8}{13} \)

17. \( P(\text{freshman or female}) = P(\text{freshman}) + P(\text{female}) - P(\text{female freshman}) = \frac{10 + 15}{50} = \frac{15 + 5}{50} = \frac{15}{50} = \frac{30}{50} = \frac{3}{5} \)

18. \( P(\text{both red}) = P(\text{red}) \cdot P(\text{red given first ball was red}) = \frac{5}{20} \cdot \frac{4}{19} = \frac{4}{19} = \frac{1}{19} \)

19. \( P(\text{all correct}) = P(\text{correct}) \cdot P(\text{correct}) \cdot P(\text{correct}) \cdot P(\text{correct}) = \left(\frac{1}{4}\right)^4 = \frac{1}{256} \)

20. number of favorable outcomes = 20, number of unfavorable outcomes = 15

Odds against being a man are 15:20, or 3:4.

21. a. Odds in favor are 4:1.  
   b. \( P(\text{win}) = \frac{4}{1} + 4 = \frac{4}{5} \)

22. \( P(\text{not brown eyes}) = \frac{18 + 10 + 20 + 12}{22 + 18 + 10 + 18 + 20 + 12} = \frac{60}{100} = \frac{3}{5} \)

23. \( P(\text{brown eyes or blue eyes}) = \frac{22 + 18 + 18 + 20}{22 + 18 + 10 + 18 + 20 + 12} = \frac{78}{100} = \frac{39}{50} \)
24. \( P(\text{female or green eyes}) = P(\text{female}) + P(\text{green eyes}) - P(\text{female and green eyes}) \)
\[ = \frac{18+20+12}{100} + \frac{10+12}{100} - \frac{12}{100} \]
\[ = \frac{50}{100} + \frac{22}{100} - \frac{12}{100} \]
\[ = \frac{60}{100} \]
\[ = \frac{3}{5} \]

25. \( P(\text{male|blue eyes}) = \frac{18}{18+20} = \frac{18}{38} = \frac{9}{19} \)

26. \( P(\text{two people with green eyes}) = P(\text{green eyes}) \cdot P(\text{green eyes|first person has green eyes}) = \frac{22}{100} \cdot \frac{9}{99} = \frac{7}{150} \)

27. \( E = 65,000(0.2) + (-15,000)(0.8) = 1000 \). This means the expected gain is $1000 for this bid.

28. \( E = (-19) \cdot \frac{10}{20} + (-18) \cdot \frac{5}{20} + (-15) \cdot \frac{3}{20} + (-10) \cdot \frac{1}{20} + (80) \cdot \frac{1}{20} \)
\[ = -\frac{190}{20} - \frac{90}{20} - \frac{45}{20} - \frac{10}{20} + \frac{80}{20} \]
\[ = -\frac{255}{20} = -12.75 \]
This expected value of $-12.75 means that a player will lose an average of $12.75 per play in the long run.